





Proceedings of the CEM Mathematics Symposium

Copyright © 2008 by the Center for Educational Measurement, Inc.

All rights reserved. Except for the quotation of short passages for the purpose of review, no part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means: electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the Center for Educational Measurement, Inc.

Published by Center for Educational Measurement, Inc. 24th Floor Cityland Pasong Tamo Tower 2210 Chino Roces Avenue, Makati City www.cem-inc.org.ph

Layout by Ernesto E. Felicen

ISSN: 1656-8575

FOREWORD

A carefully planned study of the curriculum is central to reform in education. Its findings must lead to a greater understanding of teaching and learning processes as they unfold in our classrooms. Because student development is at stake, curriculum change must be managed deliberately with practically nothing left to chance.

Knowing what students know begins with assessment. And the role of assessment vis-à-vis the curriculum is to assist teachers and curriculum designers in adapting strategies and adjusting learning targets in order to maximize student learning. When students of other countries excel in mathematics, and our own students seem to lag behind, we need to know why. Where can we find some answers? What can we do to reverse the trend?

The first of the CEM Symposia Series opens this year on the theme Emerging Issues in the Mathematics Curriculum: What we know, What we need to know. It brings together significant persons of institutions committed to improve teaching and learning in mathematics.

Dr. Yeap Ban Har's keynote paper deals with an analysis of problemsolving mathematics items of a large-scale primary school leaving test in Singapore. Mr. Jason Moseros presents a comparison of the elementary mathematics curricula of the Philippines and Singapore. Ms. Ma. Angeles Sampang examines error patterns revealed by responses to items of CEM diagnostic tests in mathematics and proposes possible directions for intervention.

It is hoped that the insights gained from these papers will serve as guides in knowing what we need to change and how to frame, manage, and assess the change for the benefit of the Filipino mathematics learner.

Finne Decenticio

LENORE DE LA LLANA-DECENTECEO President

Table of Contents

Keynote Address: An Analysis of Mathematics Items
in National Examinations in Singapore
Dr. Yeap Ban-Har1
A Comparative Analysis of the Philippine and
Singapore Elementary Mathematics Curricula
Jason V. Moseros14
Error Pattern Analysis in Mathematics:
A Springboard for Intervention
Ma. Angeles A. Sampang
Panel Reactions
Dr. Rosemarievic Villena-Diaz and Dr. Lynda Parreño
Open Forum Highlights71
List of School Participants
Symposium Program
Publication Guidelines

AN ANALYSIS OF MATHEMATICS ITEMS IN NATIONAL EXAMINATIONS IN SINGAPORE

Dr. Yeap Ban Har National Institute of Education - Singapore

This paper is based on an analysis of mathematics items in national examinations in Singapore with the purpose of identifying key constructs which are assessed. Released items from elementary school national examination papers for the last five years were sampled for analysis along with items from the specimen paper. It was found that there is substantial emphasis on problem solving. Visualization and generalization were two constructs that feature prominently in the problemsolving items. Novelty and complexity were two characteristics found in the problem-solving items. There is an emphasis on mental strategies and number sense in both the basic and problem-solving items. The discussion is extended to items from the national examination for secondary students.

Columbus was searching for hardware—precious metals, silk, and spices—the sources of wealth in his days. I was searching for software—brain power and knowledge workers—the sources of wealth in our day.

> Thomas L. Friedman, 2006 Author of The World Is Flat

In Singapore, more so than anywhere else, nurturing a pool of knowledge workers is important for the economic development of the country. The mathematics curriculum document states that "an emphasis on mathematics education will ensure that we have an increasingly competitive workforce to meet the challenges of the 21st century" (Ministry of Education of Singapore, 2006, p. 5). In line with this, a problem-solving curriculum was introduced in Singapore in 1992 (Ministry of Education of Singapore, 1990). It has been revised twice, in 2001 and in 2007, but the focus on problem solving has been retained and further emphasized. The implementation of the problem-solving curriculum seems to have a positive effect on student achievement and attitude towards mathematics. Singapore students have been known to attain a high level of achievement in mathematics (Ministry of Education of Singapore, 2004). The data from the 2003 Trends in Mathematics and Science Study (TIMSS 2003) indicated that 38% of the fourth graders and 44% of the eighth graders in Singapore achieved the advanced level in mathematics performance (in contrast to the international average of 8% and 6%, respectively). This indicates that even the average students in Singapore have a high mathematics achievement level. At the same time, it has been reported that Singapore students enjoy mathematics (Ho & Lin, 2004).

It is well known that assessment drives the curriculum. The main purpose of this paper is to describe how the problem-solving curriculum is assessed in the national examination. Through an analysis of items in national examinations in Singapore, key features and constructs of test items in such examinations are identified.

National Examinations in Singapore

The Singapore education system comprises primary education (Primary 1 to Primary 6), secondary education (Secondary 1 to Secondary 4) and post-secondary education which includes junior colleges (JC1 and JC2) as well as polytechnics and vocational schools. There are three main national examinations: (1) Primary School Leaving Examination (PSLE) taken at the end of Primary 6, (2) General Certificate of Education Ordinary Level Examination (GCE O-Level) taken at the end of Secondary 4, and (3) General Certificate of Education Advanced Level Examination (GCE A-Level) taken at the end of JC2. However, about two in five secondary students sit for the General Certificate of Education Normal Level (GCE N-Level) at the end of Secondary 4 before the majority of them proceed to study another year and before they sit for the GCE O-Level. In other words, these students complete the secondary schooling in five years, instead of four. About 30% of Singapore students study in the junior colleges and sit for the GCE A-Level.

In this study, the detailed analysis was done for the PSLE. This was done for the following reason. There was a new format for mathematics in the GCE A-Level examination at the end of 2007 and the GCE O-Level examination at the end of 2008. Analysis of previous years' items from

2

these two examinations may not give an accurate picture of any shifting emphasis. There are indications that the new examinations will place an emphasis on elements that have not been emphasized in the previous years' examinations. Although there will be a new format for mathematics for PSLE at the end of 2009, the Ministry of Education has assured the public through a press release that the emphasis and difficulty level of the revised format will be similar to examinations in recent years (Ministry of Education of Singapore, 2007a).

The Study

All released items in the PSLE in the last five years (2003 to 2007) were sampled for analysis. In addition, the specimen paper, which was published to give the public a sense of what the examination entails, was sampled for analysis. In the first stage of the analysis, the items in the specimen paper were classified according to their mark value. The items within each category were compared to identify the key competencies that are being assessed by these items. In the second stage of analysis, items in the last five years were first classified as basic or problem-solving items. All the problem-solving items were analyzed to identify the nature of such items.

It was decided that it was necessary to analyze problem-solving items across five years as such items tend to be more varied across years and not always similar to those in the specimen paper than items assessing basic knowledge. Items assessing basic knowledge tend to be similar across years and to those in the specimen paper. Hence, it was sufficient to analyze such items in the specimen paper.

It should be noted that the items sampled for analysis were from the examination that was taken by about 90% of the Primary 6 students in each cohort (Ministry of Education of Singapore, 2007b). About 10% of the Primary 6 students sat for an alternate paper. These students were assessed to have not mastered the foundation of mathematics taught in the first four grades and in their fifth and sixth grades had another opportunity to strengthen their foundation while learning selected content that the other fifth and sixth graders typically study. The majority of the students study Mathematics while the subject that focuses on the foundation knowledge is referred to as Foundation Mathematics, and not Foundation Mathematics, is

more appropriate as the competencies assessed in Foundation Mathematics form a subset of those in Mathematics.

The main research problem was to identify competencies that were assessed by items in the national examination in Singapore. The specific research questions were:

- 1. What broad competencies were assessed by items in the PSLE?
- 2. What are considered to be basic competencies?
- 3. What are the constructs assessed by and the characteristics of problem-solving items?

The Findings

The examination comprised 15 multiple-choice items of which ten were valued at one point each and five at two points each, 20 short-answer items of which ten were valued at one point each and ten at two points each; and 13 long-answer items where solution method must be communicated to earn the complete credit. In the revised format to be used from 2009 onwards, calculator is allowed for five of the two-point short-answer items and all the long-answer items. All the items analyzed were from the examinations in 2003 to 2007 and, hence, did not involve the use of calculator. There were minor variations to this format for the examinations in 2003 to 2005. Table 1 provides a summary of the item types in the examinations.

Table 1

Item Types in the PSLE Mathematics

Item Type	Value per item	Number of items	Percentage of total score
Multiple-choice items	1	10	10%
	2	5	10%
	1	10	10%
Short-answer items	2	10	20%
Long-answer items	3 or 4 or 5	13	50%

What were the broad competencies assessed by the items in the PSLE? There were three categories of broad competencies assessed by the items in the specimen paper: (1) basic skills, (2) basic application, and (3) problem solving. Comparing all the one-point items in the specimen paper suggested that basic skills included (1) knowledge of facts, (2) knowledge of concepts, (3) computation and other procedures, (4) technical skill such as measuring and reading graphs, and (5) one-step application. Table 2 gives an example of each. The ability to solve one-step word problems is considered basic.

Word problems such as,

A photocopier can print 40 copies in 30 minutes. At this rate, how many copies can it print in 1 hour and 30 minutes? (Singapore Examination and Assessment Board, 2007)

were in the category of items that were given one point each. This suggests that the ability to see that 1 hour 30 minutes is three times 30 minutes should be automatic and is not considered to be a significant step in solving this problem. Similarly, the ability to see that three sets of 70 is equal to 210 is not considered to be a significant step worthy of any credit. These and other similar analyses of one-point items suggest that the ability to perform mental computation was expected of candidates at the end of primary schooling.

Table 2

Type of basic skill	Example
Knowledge of facts	Candidates were asked to select a figure that has parallel lines from four figures. Candidates had to know parallel lines.
Knowledge of concept	Candidates were asked to select the smallest number from four decimal numbers: 0.5, 0.21, 0.038 and 0.103. Candidates had to know the concept of place value.
Computation and other procedures	Candidates were asked to find the value of 2.5 + 8.07 as well as to find the perimeter of a rectangle with given dimensions.
Measuring and reading graphs	Candidates were asked to read the scale on a weighing scale as well as to read a pie graph.
One-step application	Candidates had to solve problems such as finding the amount paid for six jars of jam if the two jars were sold for \$2.90. Candidates also had to find the value of an angle at a point when the values of the other three angles were given (70° each).

Examples of Basic Skills Assessed

An Analysis of Mathematics Items in National Examinations in Singapore Comparing all the two-point items in the specimen paper suggested that these items were used to assess basic application that may involve two or three steps. A two-step word problem typical of items in this category is shown.

Grapes are sold at \$0.65 per 100 gm at a supermarket. What is the price of 2 kg of grapes? (Singapore Examination and Assessment Board, 2007)

All other items that did not fall into either basic skill or basic application were compared and were found to have two common characteristics they were either complex or novel, sometimes both. As problem-solving items, by the definition of what constitute a problem, tend to be more varied across years than basic skill or basic application items, it was necessary to sample such items across years. Out of 246 items, 189 (77%) were released. Out of the 189 items, 48 (25%) were classified as problemsolving items.

What are the characteristics of problem-solving items? The problemsolving items were found to have two characteristics. The first one was novelty. Novel items included an idea that was not found in any other items analyzed. Figure 1 shows an item that was considered to be novel. It was the only item under analysis that involved the idea of folding.



Figure 1. A novel item that included an idea not found in other items. (Singapore Examination and Assessment Board, 2007, page 66)



Figure 2. A complex item that required the ability to handle several pieces of information simultaneously.

(Singapore Examination and Assessment Board, 2007, page 33)

The second characteristic of problem-solving items is complexity. In complex situations, there is a need to handle several pieces of information simultaneously. The item in Figure 2 required problem solvers to link the volume of water in each container to the dimensions of its base as well as the height of the water level. In some instances, the complexity was introduced via one or more unknown variables. The next problem is an example of a complex item that required the ability to handle several unknown variables.

Ahmad and Mei Ling saved \$800 altogether. One fourth (1/4) of Ahmad's savings was \$65 more than 1/5 of Mei Ling's savings. How much more money than Mei Ling did Ahmad save? (Singapore Examination and Assessment Board, 2007, page 19)

The analysis surfaced two characteristics of problem-solving items in the PSLE—novelty and complexity. The next level of analysis was to determine the constructs assessed by the problem-solving items. What are the constructs assessed by problem-solving items? A comparison of the 48 problem-solving items surfaced two constructs that were demanded repeatedly in the items. The first construct was visualization. The ability to interpret, manipulate, and transform given diagrams as well as the ability to translate nonvisual information into diagrams were required by many of the problem-solving items.

In this problem,

Mr. Lau planted 9 seedlings in a row. The seedlings were planted at the same distance apart. The distance between the first and the fourth seedlings was 12 cm. What was the distance between the first and the ninth seedlings?

(Singapore Examination and Assessment Board, 2007, page 8)

there is a need to translate nonvisual information given in the form of text into a visual representation. This is an aspect of visualization.

In this problem,

Four planks P, Q, R and S are nailed together to make a frame as shown below. Plank P has 7 holes which divide it into 8 equal parts. Plank Q has 5 holes which divide it into 6 equal parts. In the frame, the holes A, B, C and D are four corners of a rectangle.



Plank P is 240 cm long. What is the total length of Plank P and Plank Q? (Singapore Examination and Assessment Board, 2007, page 9)

there is a need to interpret a given diagram. In the next problem, there is a need to both interpret some parts of the shaded area as the difference between the area of a square and a quarter circle as well as to manipulate the diagram but moving two shaded pieces to form a square. In manipulation, one has to translate and rotate parts of the diagram in mind. The shaded figure shows a flowerbed which is formed by 1 straight line and 6 identical quarter circles.



Find the perimeter of the flowerbed.

b. Find the area of the flowerbed.

(Singapore Examination and Assessment Board, 2007, page 32)

In the process of manipulating the diagram, one sometimes transforms the diagram. Transforming a diagram is not always a result of manipulating it through translation and/or rotation.

In the next problem,

At first, the ratio of Shanti's savings to Roy's savings was 5:4. After each of them donated \$60 to charity, the ratio of Shanti's savings to Roy's savings became 13:10. What was Shanti's savings at first? (Singapore Examination and Assessment Board, 2007, page 49)

One solution method that is common in Singapore textbooks is the use of diagrams. In the solution shown (Figure 3), nonvisual information is translated into diagram and subsequently transformed into one that is useful in solving the problem.



Figure 3. The use of diagrams to solve the problems. (Singapore Examination and Assessment Board, 2007)

One construct that problem-solving items in PSLE required is visualization which comprises the ability to translate non-visual information into diagrams, to interpret diagrams, and to manipulate and transform diagrams.





An Analysis of Mathematics Items in National Examinations in Singapore Another construct assessed by the problem-solving items was the ability to make connections. In the previous problem, the ability to see the connection between the two given ratios in that particular situation is important in successful solution. In the next problem, the ability to connect between the fact that the length of the rectangle is the same as the diameter of the semicircle and that the width of the rectangle is the same as the radius of the semicircle is crucial.

In all the items, it was noticed that the computation was not tedious and could be done mentally in most instances.

In at least 50% of the examination, candidates were expected to communicate their solution methods. More credit was given for the solution method than the final answer.

Other than visualization and connections, mental computation and communication were the other constructs that were assessed in the majority of the problem-solving items selected for analysis.

Conclusion and Discussion

Based on the analysis of items in the PSLE, it was found that students in Singapore were expected to engage in novel and complex problem solving in a significant portion of the national examination at the end of primary schooling. The ability to visualize, the ability to connect, and the ability to communicate one's thoughts were the constructs that the problem-solving items were found to be assessing. In particular, tedious computation was found to be absent, suggesting an emphasis on mental computation when a calculator was not available.

Although it is beyond the scope of this paper to do a similar analysis on the items in the Grade 10 and Grade 12 national examinations, two points should be mentioned. Firstly, many 'problems' in the secondary level examination tend to be not novel. They were taught as application in the textbooks and similar 'problems' could be found across years. For example, the application of solving simultaneous equations involving one nonlinear equation and solving of quadratic equations to solve 'problems' related to the intersection of a straight line and a curve is a constant task in examinations across years. As such, these 'problems' lost its novelty and complexity effects.

An Analysis of Mathematics Items in National Examinations in Singapore However, in recent years, there had been a few novel problems included in the secondary level national examinations. This may be a sign of things to come in the future for the secondary level national examinations. Two such problems, the first one for the Grade 10 examination and the second one for the Grade 12 examination, are illustrated as examples 1 and 2, respectively.

Example 1. A novel task from the Grade 10 examination.

A fly, F, starts at the point with the position vector (i + 12j) cm and crawls across the surface with a velocity of (3i + 2j) cm s⁻¹. At the instant that the fly starts crawling, a spider, S, at the point with position vector (85i + 5j)cm, sets off across the surface with a velocity (-5i + kj) cm s⁻¹, where k is a constant. Given that the spider catches the fly, calculate the value of k.

Example 2. A novel task from the Grade 12 examination.

Four friends buy three different kinds of fruit in the market. When they get home, they cannot remember the individual price per kilogram, but three of them can remember the total amount that they each paid. The weights of the fruits and the total amounts paid are shown in the following table.

	Suresh	Fandi	Cindy	Lee Lian
Pineapple (kg)	1.15	1.20	2.15	1.30
Mangoes (kg)	0.60	0.45	0.90	0.25
Lychees (kg)	0.55	0.30	0.65	0.50
Total amount paid in \$	8.28	6.84	13.05	

Assuming that, for each variety of fruit, the price per kilogram paid by each of the friends is the same, calculate the total amount that Lee Lian paid.

(Singapore Examination and Assessment Board, 2007)

References

- Ho, A. L., & Lin, L. (2004). Worldwide trend study: Students here enjoy learning maths, science. That maybe one of reason why they have come out top in both subjects. *The Straits Times*, December 15, 2004.
- Ministry of Education of Singapore. (1990). Mathematics syllabus: Primary. Singapore: Curriculum Planning Division.

- Ministry of Education of Singapore. (2004). Singapore tops the Trends in Mathematics and Science Study (TIMSS) 2003. Retrieved from http:/ /www.moe.gov.sg/media/press/2004/pr20041214.htm
- Ministry of Education of Singapore. (2006). Primary mathematics syllabus. Singapore: Curriculum Planning and Development Division.
- Ministry of Education of Singapore. (2007a). Education statistics digest. Singapore: Planning Division.
- Ministry of Education of Singapore. (2007b). Introduction of calculators in primary five and six mathematics. Retrieved from http:// www.moe.gov.sg/media/press/2007/pr20070427.htm
- Singapore Examination and Assessment Board. (2007). Mathematics: Examination questions 2003 – 2007. Singapore: Dyna Publisher.

Dr. Yeap Ban Har is an Assistant Professor at the National Institute of Education Singapore. His teaching areas include curriculum studies at the primary and secondary levels, mathematical problem solving at the graduate level, and inservice modules on problem solving and geometrical thinking at the primary levels as well as lower primary teaching. His research interests are wide-ranging and include areas such as problem solving and problem posing, textbook studies, and lesson study. He is the principal investigator in a funded project at 11 schools on the use of word problems to engage children in sensemaking.

He is also on the editorial board of two mathematics education journals – an international journal called Mathematical Thinking and Learning published by Lawrence Erlbaum Associates, Inc. and The Mathematics Educator published by the Association of Mathematics Educators. In addition, he sits on the school boards of two international schools. He holds workshops for teachers, including preprimary teachers, in Singapore and numerous Asian countries as well as in the United Arab Emirates, South Africa, Australia, and the United States of America. He often conducts seminars for parents.

Dr. Yeap has published articles focusing on arithmetic word problems found in Singaporean and American textbooks, developing ability in mathematical problem solving, analysis of Singapore mathematics textbooks, enhancing problem solving in the middle grades, and developing algebraic thinking.

COMPARATIVE ANALYSIS OF THE PHILIPPINE AND SINGAPORE ELEMENTARY MATHEMATICS CURRICULA

Jason V. Moseros

Center for Educational Measurement, Inc.

In this paper, the 2001 Singapore Elementary Mathematics Curriculum is examined and compared with the Philippine 2002 Basic Education Curriculum, particularly on content, emphasis, and organization. Analyses include: [1] mathematical content of each grade level, [2] length of mathematical topic exposure of elementary students, and [3] elementary mathematics topics unique to each country. Curricular differences in terms of development and instructional time allocated to each subject in both countries are also discussed. Results show that the elementary mathematics content covered by the two countries are guite parallel. The basic difference lies in the inclusion of topics to be emphasized either as application or enrichment. Both countries organize topics logically; however, the Singapore curriculum does away with much overlap and has less number of expected learning outcomes than what is prescribed in the Philippine curriculum. These allow Singapore students more time to learn new topics and to take up topics more thoroughly than their Philippine counterparts.

Results of national and international surveys have shown that the Philippines' mathematics performance is below par. In the 2007 National Achievement Test (NAT)¹, the Grade 6 public school students garnered a mean percentage score of 60.29% in Mathematics which is 6.63 percentage points higher than the 2006 mean percentage score of 53.66% (Department of Education, 2008). Although these results indicate an overall improved performance in Mathematics, they still point to the fact that achievement levels in elementary mathematics still fall below standards.

¹ NAT is designed to determine what graduating students know and can do in Mathematics, Science, English, Filipino, and HEKASI/Araling Panlipunan at the end of the school year. This is carried out by the National Education Testing and Research Center (NETRC). A score of 75% and over means that the student has mastery of the subject; 50% to less than 75%, near mastery; and below 50% means low mastery (National Statistics and Coordination Board, 2008).

In the 2003 Trends in International Mathematics and Science Study (TIMSS)², the Filipino fourth graders ranked 23rd out of 25 participating countries and the second year students ranked 42nd among 46 countries (TIMSS, 2003). These results gave a clear picture of the country's state of mathematics education at a global standpoint. Apparently, a lot of work is needed if Filipino students' mathematics performance is to reach the international average level.

Purpose of the Study

Improving the quality of mathematics education as well as of the other subjects is no easy task as the problem has been traced to a number of causes which include socioeconomic factors, teacher-related factors, inadequate learning materials, among others (SEAMEO INNOTECH, 2003).

Still, the situation is far from hopeless. Taking one step at a time can bring us closer to achieving an education system that can match those of the top-ranking TIMSS countries. Perhaps giving some attention to determining how these countries have made it to the top can give us a better idea of where to begin.

In this paper, the 2001 Singapore mathematics curriculum is examined. It is compared with the Philippine 2002 Basic Education Curriculum to determine their similarities and differences in terms of content, emphasis, and organization. This gives us a better impression of whether or not we are giving more than what is essential for students to learn at their level at an international perspective. Although Singapore already has a revised syllabus for the year 2007, the Singapore 2001 Primary Mathematics Syllabus is used in this study as this was the one implemented when the country attained its latest (2003) TIMSS ranking.

Singapore was chosen over all the other countries for three main reasons:

 Like the Philippine, it determines and issues curriculum guidelines at the national level;

² TIMSS is a large-scale international study of mathematics and science at grades 4 and 8 which is designed to measure trends in students' mathematics and science achievement in four-year cycles (since 1995) and carried out by the International Association for the Evaluation of Educational Achievement (IEA).

- (2) Singapore also has a basic education system similar to that of the Philippines; and
- (3) It held the topmost spot in the 1999 and 2003 TIMSS mathematics performance ranking.

Curriculum Development

Locus of Control Over Curriculum Development

Philippines

The Philippine education system is decentralized. The central/national office is engaged in policy formulation; while the regional and division offices are the implementing bodies. Supervision of schools is done at the regional and subregional levels (Mariñas & Ditapat, 2000).

The Central Office Bureau of Elementary and Secondary Education, Curriculum Development Division is responsible for the development of the basic education curriculum in the Philippines. This bureau defines the learning competencies for the different subject areas; conceptualizes the structure of the curriculum; and formulates national curricular policies. These functions are exercised in consultation with other agencies and sectors of society (e.g., industry, social and civic groups, teacher-training institutions, professional organizations, school administrators, parents, students, etc.). The subject offerings, credit points, and time allotments for the different subject areas are also determined at the national level (Mariñas & Ditapat, 2000).

Singapore

In Singapore, the Ministry of Education (MOE) has overall responsibility for the curriculum. It determines the educational structure, national goals for education, and the education program for the whole country. However, some autonomy is given to schools to take greater control over the planning and delivery of instructional programs and the adoption of teaching methods to meet the needs, abilities, and interests of their students. This is done within the framework of the parameters and guidelines defined by the MOE (INCA, 2002).

Curriculum Design

Philippines

The approach to curriculum design in the Philippines is based on content topic and competency. The Department of Education (DepED) prescribes competencies for the subject areas in all the grade/year levels. The Bureau of Elementary and Secondary Education develops, publishes and disseminates these learning competencies to the field. Most of the subject/learning areas have a list of learning competencies expected to be mastered by the students at the end of each grade/year level and also at the end of elementary/secondary schooling. Some subject/learning areas have a combination of both (i.e., learning competencies under each content/ topic). The curriculum is designed to be interpreted by teachers and implemented with variations. Schools are encouraged to innovate, enrich or adapt, as long as they meet the basic requirements of the curriculum (Mariñas & Ditapat, 2000).

The curriculum plan (learning competencies) does not present teaching methods and learning activities that teachers must follow in implementing the curriculum. The guiding philosophy is that the creativity of teachers is stimulated by the option to plan and use the appropriate teaching/learning activities independently. However, teacher's manuals or guides incorporate higher-level content areas and suggestions for teaching and assessing (Mariñas & Ditapat, 2000).

Singapore

In Singapore, the MOE sets out the policy objectives in teaching and learning of the various subjects in the curriculum and designs the subject syllabi. For each subject, a syllabus outlines in detail the rationale and specific objectives for teaching the subject at the primary level. These are accompanied by the curriculum framework, in which the lists of content topics are integrated across each grade/year level. In addition, guidelines and suggestions on the methods of teaching are highlighted, together with a clear statement of the intended standards of achievement. The syllabus then concludes with a suggested list of textbooks and available instructional resources. The MOE is also responsible for the ongoing review and systematic revision of its national curriculum (INCA, 2002).

Curricular Review and Reform

Philippines

It was only in 2002 when DepED restructured the 1983 Elementary School Curriculum and the 1989 Secondary Education Curriculum and came up with the 2002 Basic Education Curriculum. DepED assures the continuous review and refinement of the curriculum to ensure that it responds to changing needs and demands.

Singapore

This is in sharp contrast to Singapore's curricular review timetable which holds curriculum planning and review on a regular basis. In the past, MOE reexamines the curriculum every 8 to 10 years, but this has been reduced to a six-year cycle to better ensure that the curriculum is responsive to students' needs, abilities and interests; future-oriented; and economically relevant (INCA, 2002). Just recently, the country implemented the 2007 Primary Mathematics Syllabus which is a revision of the 2001 version. The new syllabus reflects the recent developments and trends in Mathematics education (Ministry of Education of Singapore, 2006a).

Academic Term

Philippines

The Philippine school year runs for 10 months or at least 200 days beginning between the first and third weeks of June until the last week of March. The academic year for elementary and high school is divided into quarters consisting of about two and one-half months. There is a one-week break between the second and third quarters. Christmas break usually begins in the third week of December, and classes resume the Monday after New Year's Day. Commencement ceremonies are often held in late March or early April. The summer break (end of fourth quarter) lasts for about two months, from the first week of April to the last week of May.

Singapore

Similarly, a school year in Singapore consists of four 10-week terms beginning on the 2rd day of January each year up to the second week of November. There is a one-week vacation after the first and third terms, a four-week holiday after the second term and six weeks at year-end. Of the 40 teaching weeks for each school year, examinations and other school activities may account for about four weeks of instructional time (INCA, 2002).

Basic Education System

Philippines

Basic education in the Philippines involves six years (Grade 1 to Grade 6) of compulsory elementary education and four years (First Year to Fourth Year) of secondary education.

Filipino students, on the average, finish elementary school at age 12 or 13 years and secondary school at age 16 or 17 years. Afterwards, they may enroll in tertiary institutions to obtain a degree or a certificate in a course of their choice. Tertiary education includes two-year post-secondary technical and vocational courses, various professional courses, and general higher education, including graduate and post-graduate studies (for students aged 17–25). Normally, a baccalaureate degree takes four years. Graduate and postgraduate courses normally take two to three years to complete (see Appendix: Figure A).

Singapore

Singapore pupils begin formal education at primary schools, starting from Primary 1 (at age 6 or 7) through Primary 6 (at age 11 or 12) which is compulsory. Afterwards, pupils who pass the Primary 6 Leaving Examination (PSLE) will progress to secondary schools. Depending on the results of the examination, pupils are placed in one of three secondary courses: the Express Course, Normal (Academic) Course, and Normal (Technical) Course. The Express Course is a four-year secondary course (Secondary 1 to Secondary 4) which leads to the Singapore-Cambridge General Certificate of Education 'Ordinary' (GCE 'O') Level Examination at the end of the course. The two Normal Courses are four-to five-year courses (Secondary 1 to Secondary 5) which leads to the Singapore –

Cambridge General Certificate of Education Normal (GCE 'N') Level Examination at the end of the fourth year. Students who do well in this examination will proceed to a fifth year of secondary education and take the GCE 'O' Level Examination at the end of the fifth year. Those who do not qualify to enter the fifth year may take up technical-vocational education and training at the Institute of Technical Education (ITE). Those who pass the GCE 'O' Level Examination will then have to compete for admission to either a Junior College (2 years), a Polytechnic (3 or 4 years) or a Pre-University Center (3 years).

Finally, students who pass the Singapore–Cambridge General Certificate of Education 'Advanced' (GCE 'A') Level Examination at the end of Junior College Year 2 or Pre-University Year 3, and students with excellent results at the end of Polytechnic Year 3 or 4 will then have to compete for admission to a local university. This is analogous to the college or tertiary (undergraduate) level in the Philippines (Ministry of Education, Singapore, 2008). (see Appendix: Figure B).

Structure of the Elementary Curriculum

Philippines

In the Philippines, all elementary (Grades 1 to 6) pupils follow a common curriculum for all subjects. However, while the curriculum implementation guidelines are issued at the national level, the actual implementation is left to schoolteachers. The schoolteachers determine the resources to be used, teaching and assessment strategies, and other processes. Furthermore, schools are given the option to modify the national curriculum to suit local contexts. Variations may be seen in terms of content sequence, teaching strategies, or other cocurricular activities that could further enhance learning. In fact, the country's Department of Education does not discourage such modifications as long as the basic requirements of the curriculum are fulfilled (Mariñas & Ditapat, 2000).

Singapore

On the other hand, Singapore pupils at the primary level go through a 4year foundation stage, from Primary 1 to 4, and a 2-year orientation stage, Primary 5 and 6.

Only pupils at the foundation stage follow a common curriculum which includes Mathematics among other subjects. At the end of the fourth grade, each pupil takes a series of tests³ for all subjects, English Language, Mother Tongue Language, Mathematics, and Science. Based on the results of these tests, each pupil is placed in one of three language streams for his upper primary years (orientation stage). A pupil who score well in these examinations is able to opt for a higher stream, EM1, while average faring pupils are sorted into EM2. Pupils who do not score well in these examinations are channeled into the EM3 stream, where they take up subjects at a lower level of study (e.g., pupils are taught foundation mathematics and basic languages). The mathematics subject offered in EM2 is at the same level of academic depth as that offered in EM1⁴.

In 2004, Singapore's Ministry of Education merged the EM1 and EM2 streams into a single stream (EM1/EM2) and schools were given the freedom to decide on how to best sort their pupils by ability⁵ (Ministry of Education of Singapore, 2007).

Content Areas in the Mathematics Curriculum

The Philippine 2002 Basic Education Curriculum (Elementary Level)

In the Philippines, all elementary students (Grades 1 to 6) are required to take the mathematics subject. They are expected to have understood and mastered all the content areas covered in their respective grade levels before advancing to higher grade levels (see Table 1).

The aim of the curriculum is to enable students to demonstrate their understanding and skills in computing with considerable speed and accuracy, estimating, communicating, thinking analytically and critically, and in solving problems in daily life using appropriate technology.

³ This is called the Primary 4 Streaming Exercise.

⁴ The only difference between EM1 and EM2 is that EM1 pupils take up Higher Mother Tongue (HMT) language in addition to the usual subjects (English, Mathematics, and Science), while EM2 pupils do not.

⁵ Among other things, schools can also exercise freedom in recommending pupils to take up HMT.

Comparative Analysis of the Philippine and Singapore Elementary Mathematics Curricula

Mathematics in Grades 1 and 2 include the study of whole numbers, addition and subtraction, basic facts of multiplication and division, basics of geometry, fractions, metric and local measurements, the use of money and their application to practical problems based on real life activities.

Table 1

Content Areas in the Philippine Elementary Mathematics Curriculum

Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
1. Whole numbers	1.Whole numbers	1. Whole numbers	1.Whole numbers	1. Whole numbers	1.Whole numbers
2.Rational numbers	2.Rational numbers	2.Rational numbers	2.Rational numbers	2. Rational numbers	2.Rational numbers
	3.Geometry	3.Geometry	3.Geometry	3. Geometry	3. Geometry
3. Measurement	4. Measurement	4. Measurement	4. Measurement	4. Measurement	4. Measurement
		5.Graphs	5.Graphs	5.Graphs	5. Graphs
		6.Calculator			

On the other hand, Grades 3 and 4 Mathematics deal with the study of whole numbers, the four fundamental operations, fractions and decimals including money, angles, plane figures, measurement and graphs.

For Grades 5 and 6, the child is expected to master the four fundamental operations of whole numbers, perform skills in decimals and fractions, conceptualize the meaning of ratio and proportion, percent, integers, simple probability, polygons, spatial figures, measurement and graphs. Simple concepts in Algebra are also introduced but further articulation is done in high school (Department of Education, 2007).

The Singapore 2001 Primary Mathematics Syllabus

Singapore primary students are required to take the Mathematics subject and to follow a common curriculum for the first four years of their formal schooling (see Table 2). The primary aim of the whole mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving.

The topics covered in primary levels 1 to 4 include addition, subtraction, multiplication, and division of whole numbers and money, the concept of angles, basic shapes, lines, and curves. Pupils are also taught how to tell time and how to convert one unit of measure to another (e.g., centimeter to meters). In the upper primary levels, the pupils are required to follow one of two prescribed Mathematics curricula depending on the results of their streaming examination. Table 2 presents the two streams or curricula for Primary 5 and 6, namely EM1/EM2 and EM3. Since the EM3 stream is intended for pupils who are less able to cope with Mathematics, it has less number of content areas and covers less complex topics than the EM1/EM2 stream.

Generally, Primary 5 and 6 pupils are taught to recognize more complex shapes, such as rhombus and trapezium, and to know the properties of three-dimensional shapes such as the cube and sphere. Pupils are also taught to read and interpret line graphs, pie charts, and bar graphs as well as to perform operations involving fractions and decimals.

Table 2

Content Areas in the Singapore Elementary Mathematic	es Curriculum	
--	---------------	--

	FOUND	ATION STAGE		ORIENTATION STAT	GE: EM1/2 STREAM
Primary I	Primary 2	Primary 3	Primary 4	Primary 5	Primary 6
1. Whole numbers	 Whole numbers Fractions 	 Whole numbers Fractions 	 Whole numbers Fractions Decimals 	 Whole numbers Fractions Decimals 	
2. Geometry	3. Geometry	3. Geometry	4. Geometry	4. Geometry	1. Geometry
 Money and Measures 	 Money and Measures 	 Money and Measures 	 Money and Measures 	5. Mensuration	2. Mensuration
4. Statistics	5. Statistics	5. Statistics	6. Statistics	6. Statistics	3. Statistics
				7. Average, Rate, and Speed	4. Average, Rate, and Speed
				8. Ratio and	5. Ratio and
				Proportion.	Proportion.
				9, Percentages	6. Percentages
					7. Algebra
				ORIENTATION ST	AGE: EM3 STREAM
				Primary 5	Primary 6
				 Whole numbers Practions Decimals Geometry Money, Measures and Mensuration Statistics 	 Geometry Money, Measure and Mensuration Statistics Average and Rah
					5. Direct propartion 6. Percentage

Methods and Procedures

In this study, the Philippine 2002 Basic Education Curriculum is compared with the 2001 Singapore Primary Mathematics Syllabus mainly on content coverage, emphasis, sequence, and duration. The number of minutes per week allocated to each subject per grade level and how each curriculum addresses the needs of slower mathematics students are also discussed.

In identifying the content areas given more emphasis in the Philippines than in Singapore (and vice versa) and mathematical content unique to each country, a simple matching of the two syllabi was done. However, in identifying mathematical topics emphasized and introduced at earlier grade levels in the Philippines than in Singapore (and vice versa) and in determining the length of mathematical content exposure of elementary students in both countries, a topic chart or checklist was constructed to facilitate the matching and tallying of topics.

In the analysis on the length of mathematical content exposure of elementary students in the Philippines and Singapore, five values are reported, namely, total number of topics, average number of grade levels per topic and its corresponding Philippine/Singapore ratio, average number of topics per grade level and its Philippine/Singapore ratio.

In counting the total number of topics, each topic is counted as one regardless of the number of times it appears across grade levels. However, in determining the number of topics per grade level, topic frequency was taken into account.

The Philippine/Singapore (P/S) Ratio is obtained by dividing the Philippine average by that of Singapore. The resulting value gives the number of times the Philippines exceeds or is below Singapore in terms of the aspect being compared. A value of 1.0 indicates that both countries have roughly the same average. A P/S Ratio that is greater than 1.0, say 1.6, means that the Philippine average is 1.6 times that of Singapore or that the Philippine average is 60% more than that of Singapore. A value that is less than 1.0, say 0.6, means that the Philippine average is 0.6 times that of Singapore or that the Philippine average is 40% less than that of Singapore. This interpretation applies to all sections where P/S ratios are used; namely, the analyses on time allocation, topic exposure, and number of learning competencies per grade level.

Since this paper reports findings on several analyses, some of the procedures done are mentioned as results are discussed in the different sections.

Analysis of the Philippines and Singapore Elementary Mathematics Curricula

Addressing the Needs of the Slower Students

The Philippines and Singapore both recognize the fact that students learn at different speed. The difference lies in the way the needs of the slower mathematics students are addressed.

Philippines

All elementary students in the Philippines follow a common curriculum for all subjects. The curriculum is designed to be interpreted by teachers and implemented with variations as long as its basic requirements are met. Teachers are given the freedom to plan and use the teaching method, resources, and assessment strategies they deem appropriate to the level of their students' ability. Teachers may use a different teaching strategy for students who are having a hard time in mathematics. Some employ peer teaching which involves one or more students teaching other students in a particular topic. In most cases, teachers place their slow students in remedial classes where they can be given special assistance.

Singapore

In Singapore, the slower mathematics students are taught according to a special curriculum for Primary 5 and 6. Based on the results of a school's own Primary 4 year-end examination, slower students are assigned EM3 while the average and fast learners are assigned EM1/2. Schools identify students who will benefit from the EM3 curriculum for Primary 5 and 6. Although parents have the final say on which stream their child goes into, they almost always follow the school's advice on these matters (Ministry of Education of Singapore, 2007).

The EM3 curriculum includes review of topics covered during the foundation stage (Primary 1 to 4) as well as topics covered in EM1/2 but at a slower pace. For instance, some EM1/2 (normal stream) Primary 5 topics like geometrical construction, average and rate, and percentages are

not introduced in EM3 Primary 5 but are delayed until Primary 6. This EM3 structure gives room for the review of previously taken topics in the foundation stage without really compromising new topics that Primary 5 and 6 students ought to be learning at their grade level.

Table 3 shows the proportion of EM3 learning outcomes seen in other primary levels. Notice that in EM3 Primary 5, 42% of its learning outcomes are but a repetition of some Primary 4 outcomes and only about 18% are outcomes similar to those in the Primary 5 EM1/2 stream. Approximately 55% of mathematical content covered in EM3 Primary 5 is a review of the subject matter, and only about 45% is newly introduced content. A similar observation can be made on the EM3 Primary 6 level. Roughly 35% of learning outcomes in this level are from the EM1/2 Primary 5 level, and only about 16% are the same as those in the normal stream for Primary 6. Notice also that, contrary to the Primary 5 EM3 level, the mathematical content in Primary 6 concentrates on Primary 5 subject matter and less on Primary 4. The EM3 framework, thus, organizes mathematical content in such a way that content covered in Primary 5 does not overlap with what is taken up in Primary 6. Care has been taken in the assignment of mathematical content for Primary 5 and 6 so as not to spend too much time on the review of previously covered content which would consequently reduce the time for learning new material.

Other animary lawala	Percentage of recurring EM3 learning outcomes			
Other primary levels -	Primary 5 (%)	Primary 6 (%)		
1	0.0	0.0		
2	3.0	0.0		
3	10.0	0.0		
4	42.0	3.0		
5 (EM1/2)	18.0	35.0		
6 (EM1/2)	0.0	16.0		

Proportion of EM3 Learning Outcomes Seen in Other Primary Levels

Note. None of the learning outcomes in EM3 Primary 5 is repeated in EM3 Primary 6.

Time Allocation

Table 4 shows the proportion of total instructional time per week allocated to each subject in the Philippine and Singapore elementary mathematics curriculum. In Grades 1 to 4, both countries allocate the longest time to the English subject, with the Philippines allocating 28% of its total instructional time and Singapore allocating as much as 32%. Mathematics and Filipino (for the Philippines) or Mother Tongue (for Singapore) comes next each with 22% for the Philippine education and 20% and 26%, respectively, for that of Singapore. In Grades 5 to 6, both countries allocate the longest time to other subjects (true only for Singapore's EM1/2 stream) followed by English, then Mathematics or Filipino/Mother Tongue. Singapore's EM3 stream, however, still allots the longest time to English

Table 4

Proportion of Total Instructional Time per Week Allocated to Each Subject in the Philippine and Singapore Elementary Mathematics Curricula

Subject	Philip	pines	Singapore			
	Condex 1444	Cardon S and 6	Code La La	Grades 5 and 6		
	Grades 1 to 4	Grades 5 and 6	Grades 1 to 4	EM1/2	EM3	
Mathematics	22%	16%	20%	19%	27%	
English	28%	21%	32%	26%	33%	
Science	7%	16%	7%	10%	6%	
Filipino/Mother tongue	22%	16%	26%	18%	8%	
Other subjects	21%	31%	15%	27%	26%	

The number of minutes allocated to each subject per week in each grade level of the Philippine and Singapore elementary curricula are shown in Tables 5 and 6, respectively. Comparing the two tables, it can be seen that the Philippines allots the same amount of time to Mathematics and Filipino across grade levels. On the other hand, Singapore allocates more time to Chinese/Malay/Tamil than Mathematics in Grades 1 to 3 but does otherwise in Grades 5 and 6. It can also be noticed that both countries do not have Science for Grades 1 and 2.

Table 5

Philippine Elementary	Education: Weekly Lesson Timetable
(2002 Basic Education	Curriculum)

Subject	Weekly Time Allocated to each Subject (in minutes)						
	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	
Mathematics	400	400	400	300	300	300	
English language	500	500	500	400	400	400	
Science			200	300	300	300	
Filipino language	400	400	400	300	300	300	
Makabayan	300	300	300	500	600	600	
Total weekly minutes	1600	1600	1800	1800	1900	1900	

Source: Department of Education, 2002

Table 6

Singapore Elementary Education: Weekly Lesson Timetable

Subject	Weekly Time Allocated to each Subject (in minutes)							
	Grade 1	Grade 2	Grade 3	Conda 4	Grades 5 and 6			
	Grade I	Ofauc 2	Grade 5	Grade 4	EM1/2 285	EM3		
Mathematics	210	270	330	330	285	390		
English	510	510	450	390	375	480		
Science			90	120	150	90		
Chinese/Malay/Tamil	450	390	360	330	270	120		
Other subjects	270	270	270	300	390	390		
Total weekly minutes	1440	1440	1500	1470	1470	1470		

The Philippine and Singapore average weekly time allocations for each subject across grade levels were computed for purposes of comparison (see Table 7). As indicated by the P/S Ratios, the Philippines, generally, allocates more time to mathematics instruction per week than Singapore in either the EM1/2 or the EM3 stream. The Philippines has 23% more instructional time for mathematics than what is allocated in the EM1/2 stream and 14% more instructional time than what is prescribed in the EM3 stream. In sum, the Philippines has 23% more total weekly minutes than Singapore in the elementary level.

Table 7

Subject	Average Numbe Subject per V	P/S _[EM1/2]	P/S _(EM3)		
	01/11 - 1	Singa	pore	Ratio	Ratio
	Philippines	EM1/2	EM3		
Mathematics English	350 450	285 447	306 468	1.23 1.01	1.14 0.96
Science	275	120	100	2.29	2.75
Filipino/Mother Tongue	350	360	330	0.97	1.06
Other Subjects	433	300	300	1.44	1.44
Total weekly minutes	1858	1512	1504	1.23	1.23

Elementary Education in the Philippines and Singapore: Average Weekly Time Allocation of Each Subject

Content Areas Emphasized and Introduced at Each Grade Level

There is no solid basis on which emphasis of each content area can be determined as no information is available on the time allocated to each topic per school year and no classroom observation was done. Assumptions were made based on the duration of topic coverage across grade levels.

General Content Areas

Both countries cover four main areas, namely, Numbers, Geometry, Measurement, and Statistics. Numbers is broken down into Fractions, Decimals, Ratio and Proportion, and Percentage. The Philippine content area Graphs and the Singapore strand Statistics is considered as the same area for both deal with graphs and charts. The topic Money is dispersed in different content areas in the Philippines, while Singapore treats it as a special content area that is paired with Measures.

The Philippine curriculum reflects the need to cover the areas Probability and Calculator in the elementary level, while that of Singapore prioritizes Algebra and Average, Rate, and Speed instead.

As shown in Figure 1, the Singapore curriculum greatly emphasizes three content areas, namely, Geometry, Measurement (or Money and Measures), and Statistics as these areas are covered in all six grade levels. The content areas Whole Numbers and Fractions come next, being covered in five and four grade levels, respectively.

Comparing Singapore's two language streams, EM1/2 and EM3, it can be noticed that the EM1/2 stream covers more content areas than the EM3 stream. In grade 5, the content areas Ratio / Proportion, Percentages, and Average, Rate, and Speed are prescribed in the EM1/2 stream but not in EM3. In grade 6, pupils take up Algebra in EM1/2 but not in EM3.

The Philippine curriculum puts great emphasis on the areas Whole Numbers, Fractions, and Measurement, taking them up from grade 1 to grade 6, but gives less emphasis on Geometry and Statistics as these are taken up only in five and four grade levels, respectively.

Furthermore, the Philippine curriculum also gives more emphasis on the areas Fractions and Decimals than the curriculum of Singapore. The Philippine curriculum prescribes these areas in more grade levels than the Singapore curriculum.

It can also be seen from Figure 1 that the Philippines introduces Fractions earlier in the curriculum than Singapore. The area is introduced as early as grade 1 in the Philippines while Singapore takes it up in grade 2. On the other hand, Singapore introduces Geometry and Statistics earlier than the Philippines. Singapore introduces these areas as early as grade 1 while the Philippines introduces Geometry in grade 2 and Statistics in grade 3.

Specific Content Areas

Figure 2 shows in detail the mathematical content introduced at earlier grade levels in the Philippines than in Singapore and vice versa. The content areas and mathematical topics presented are those that are common to both countries.

Since the EM3 stream is for slower mathematics students, perhaps it is safe to assume that all topics covered in the EM3 stream are deemed essential by Singapore and outweigh excluded topics that are covered in the EM1/2 stream. If this were true, then the topics enumerated in Figure 2 are the essentials for Singapore except for tessellation, and area and circumference of circles.

CONTENT AREAS	GRADE LEVELS								
	1	2	3	4	5	6			
Whole Numbers									
Fractions						A Real Property lies:			
Decimals					-				
Geometry									
Measurement									
Statistics									
Ratio / Proportion									
Percentage									
Algebra Average / Rate / Speed									
Probability						1			
Calculator			and the second second		1	_			

Legend:

Philippine Curriculum Singapore EM1/2 Stream Singapore EM3 Stream

Figure 1. Elementary mathematics content coverage across grade levels in the Philippines and Singapore: content areas.

Whole numbers, fractions, and decimals. It can also be noticed from Figure 2 that Singapore seems to emphasize the need to teach addition and subtraction early in the curriculum. The concept and use of addition and subtraction in the content areas Whole Numbers, Fractions, and Decimals are taught the first time these areas are taken up. Whole Numbers are introduced in grade 1, Fractions in grade 2, and Decimals in grade 4; in these grade levels, computation using addition and subtraction is also covered. In the Philippines, this is only true for Whole Numbers and Decimals as the teaching of addition and subtraction of fractions is delayed a year after Fractions are introduced.

CONTENT AREA/COMMON TOPICS	GRADE LEVELS						
CONTENT AREA/COMMON TOPICS	1	2	3	4	5	6	
WHOLE NUMBERS							
1. number notation and place values	Relation of the	The second second					
2. cardinal/ordinal numbers							
3. additionisubtraction		- Constant					
4. multiplication/division	1200	No.		and the second			
5. odd and even numbers							
6. approximation/estimation							
7. order of operations	-		-				
8. comparing and ordering	and real	10000					
9. factors and multiples							
FRACTIONS					1		
10. concept of fractions				T			
11. equivalent fractions							
12. comparing and ordering							
13. addition/subtraction	-						
14. mixed numbers/improper fractions			1				
15. multiplication							
16. division							

Legend:

Philippine Curriculum Singapore EM1/2 Stream Singapore EM3 Stream

Figure 2A. Elementary mathematics content coverage across grade levels in the Philippines and Singapore: common topics—whole numbers & fractions.

Comparative Analysis of the Philippine and Singapore Elementary Mathematics Curricula

32
CONTENT AREA/COMMON TOPICS			GRAD	E LEVELS		
CONTENT AREAGOMMON TUPICS	1	2	3	4	5	6
DECIMALS						
17. number notation and place value						
18. addition/subtraction						
19. multiplication/division				Constanting of		
20. conversion between decimals and fractions				1.54 CARGO	-	12 12 12
21. approximation and estimation						
22. comparing and ordering			-	100.57000		
GEOMETRY						
23. shapes/geometrical figures						
24. tessellation						
25. symmetry		20202				
26. geometrical construction						
27. perpendicular and parallel lines						
28. concept of angles						
29. 2-D representation of a 3-D solid						

Legend:

Philippine Curriculum Singapore EM1/2 Stream Singapore EM3 Stream

Figure 2B. Elementary mathematics content coverage across grade levels in the Philippines and Singapore: common topics - decimals & geometry.

CONTENT AREACOMMON TOPICS	1.		GRAD	E LEVELS		
CONTENT AREA COMMON TOPICS	1	2	3	4	5	6
REASUREMENT						
30. measurement of length						
31. measurement of mass						
32. measurement of volume						
33. measurement of time (12-hour)						
34. money	3. m.					
35. area of square						
36. area of rectangle						
37. area of circle						
38. area of triangle						
39. area of other figures						
40. perimeter of square						
41. perimeter of rectangle						
42. perimeter of triangle						
43. perimeter of other figures						
44. circumference of circle						

Legend:

Philippine Curriculum Singapore EM1/2 Stream Singapore EM3 Stream

Figure 2C. Elementary mathematics content coverage across grade levels in the Philippines and Singapore: common topics - measurement.

Both countries seem to recognize the difficulty of learning the topic multiplication and division of fractions as these topics are taken up two to four years after fractions are introduced. Fractions are introduced as early as grade 1 in the Philippines and grade 2 in Singapore but the topic multiplication of fractions is delayed until grade 4, while division of fractions is delayed until grade 4 in the Philippines and until grade 5 in Singapore.

Measurement. The Philippine curriculum seems to give emphasis to the topics volume and area of square and rectangle as these are taken up earlier and covered in more grade levels than what is prescribed in

CONTENT AREA/COMMON TOPICS	GRADE LEVELS						
CONTENT AREACCOMMON TOPICS	1	2	3	4	5	6	
STATISTICS					940		
45. picture graphs (w), w/o scales)							
46. bar graphs							
47. line graphs							
48. pie charts						a nana	
AVERAGE, RATE, AND SPEED							
49. average							
RATIO AND DIRECT PROPORTION							
50. ratio and direct proportion							
PERCENTAGE					-		
51. percent							

Legend:

Philippine Curriculum Singapore EM1/2 Stream Singapore EM3 Stream

Figure 2D. Elementary mathematics content coverage across grade levels in the Philippines and Singapore: common topics - statistics, average rate and speed, ratio and direct proportion, and percentage.

Singapore. On the other hand, the Singapore curriculum gives emphasis to the perimeter of squares and rectangles as these are introduced earlier and are covered in more grade levels than that of the Philippines.

Statistics. In the area of Statistics, Singapore seems to underscore the need to teach picture graphs and bar graphs. These topics are introduced earlier and are covered in more grade levels than what is done in the Philippines.

Topic Exposure

Both the Philippine and the Singapore curricula present mathematical content per grade level organized into strands or content areas (e.g., Whole Numbers) which are further subdivided into topics (e.g., Number Notation and Place Values). Listed under each topic are specific learning outcomes or learning competencies (e.g., read and write numbers up to 100 in numerals and in words) which students are expected to acquire at a particular grade level.

Prior to examining the mathematical content covered in both countries, it is necessary to come up with a uniform topic classification to facilitate the matching and tallying of topics and learning competencies. For this purpose, the Singapore topic organization was generally followed, as shown in the country's Primary Mathematics Syllabus. Each Philippine learning competency was reclassified following Singapore's topic categories. In cases where a learning competency does not fall under any Singapore topic, its original (Philippine) topic classification was retained. Some closely related Singapore topics were merged to form one topic (e.g., money, addition and subtraction of money, and multiplication and division of money were merged and classified as money) and some were renamed or modified to facilitate the classification of topics in both countries.

Topic repetition is one way of aiding retention. The more frequently the students are presented with the same topics, the better they can absorb what they are taught. However, too much repetition may consume valuable instructional time that could have been spent on learning other topics. Table 8 presents the Mathematics content exposure of elementary students in the Philippines and in Singapore. In determining the total number of topics across grade levels, each topic is counted as one even if it appears in two or more grade levels. The average number of grade levels per topic is an indicator of the length of exposure of elementary students to each mathematics topic. It indicates the average number of grade levels that cover the same topic. The higher the average, the longer the students cover the given topic.

Table 8

Curriculum	Total number of topics	Average 1 grade levels	number of per topic	Average num per grad	CONTRACTOR OF A DESCRIPTION OF A DESCRIP
	or topics	Average	*P/S Ratio	Average	P/S Ratio
Philippines	69	2.1	1.1	25.8	1.3
Singapore (EM1/2)	60	1.9		19.7	
Philippines	69	2.1	0.9	25.8	1.2
Singapore (EM3)	52	2.4		21.0	

Comparative Analysis: Mathematics Topic Exposure of Grades 1 to 6 Students in the Philippines and Singapore

*Philippines over Singapore Ratio

Total Number of Topics

Results show that the Philippines has about as much as 15 to 33 percent more topics than Singapore in the whole elementary mathematics curriculum. Approximately, Philippine students cover a total of 69 topics while Singapore students cover a total of 60 topics in the EM1/2 stream and 52 in the EM3 stream.

Average Number of Grade Levels per Topic

Counting the average number of grade levels per topic, the results indicate that Philippine students dwell on topics longer than Singapore students do in the normal stream (EM1/2). However, against students in the EM3 stream, the Philippine average topic duration is relatively shorter.

The Philippine average duration is 2.1 grade levels per topic, while those of Singapore are 1.9 for the EM1/2 stream and 2.4 for the EM3 stream. These signify that the Philippine curriculum repeats topics 10 percent more than the Singapore EM1/2 stream and 10 percent less than the Singapore EM3 stream.

Average Number of Topics per Grade Level

Examining the average number of topics per grade level, the figures indicate that topics covered in the Philippines per grade level outnumber those of Singapore in either language stream. It can be seen in Table 8 that the Philippine average number of topics per grade level is approximately 25.8, while those for Singapore are 19.7 and 21.0 for the EM1/2 and EM3 streams, respectively. These figures indicate that students from the Philippines cover about 20 to 30 percent more topics per grade level than their Singaporean counterparts.

Comparing Singapore's two language streams, it is interesting to note that the average number of topics per grade level in the EM3 stream is greater than that in the EM1/2 stream. This appears to contradict the findings on the streams' total number of topics (also in Table 8) where the EM1/2 topics outnumber those of EM3. In terms of the total number of topics, the EM1/2 stream has 15 percent more topics than EM3; while in terms of the average number of topics per grade level, the EM3 stream outnumber EM1/2 by approximately 7 percent. The obvious reason for this is topic repetition. Since topics here are counted as many times as they appear across grade levels, the quantity of EM3 stream's repeating topics added significantly to the total number of topics that students cover in each grade level. Since the EM3 stream is meant for students who are behind in mathematics, it is reasonable to expect more topic repetitions here than in the EM1/2 stream.

Total Number of Learning Competencies per Grade Level

Table 9 shows the comparison between the total number of learning competencies or learning outcomes per grade level in the Philippines and in Singapore. It is interesting to note that although Singapore's EM3 stream is meant for students less able to cope with mathematics, this stream has more learning competencies than the EM1/2 stream in both Grades 5 and 6, which is for average to advanced learners. Clearly, more is expected of the slow learners in terms of knowledge acquisition and skill enhancement than those in the normal stream. This makes sense as EM3 students have a lot of catching up to do on basic concepts and foundation skills.

Table 9

Grade	Number of learn	ing competen	cies	P/S R	atio
level	Philippines	Singapore		1/5 K	auo
1	65	35		1.	9
2	103	32		3.	2
3	90	35		2.6	
4	130	55		2.	4
		EM1/2	EM3	EM1/2	EM3
5	108	49	67	2.2	1.6
6	150	26	31	5.8	4.8

Comparative Analysis: Total Number of Elementary Mathematics Learning Competencies per Grade Level in the Philippines and in Singapore

It is noticeable that in all grade levels, the number of Philippine learning competencies per grade level exceeds that of Singapore. There are almost two to six times more learning competencies in the Philippine curriculum than that in Singapore. As indicated by the P/S ratios, the Philippine learning competencies in grades 1 to 4 are 1.9 to 3.2 times Singapore's number. In grades 5 and 6, Philippine learning competencies outnumber that of Singapore's EM1/2 stream by as much as 480%, and the EM3 stream, by as much as 380%.

In both countries, there is no clear trend as to the number of learning competencies per grade level as topics may vary from one grade level to the next. In the Philippines, the Grade 1 curriculum has the fewest prescribed learning competencies while the most number is in Grade 6. In contrast, Singapore's Grade 6 curriculum appears to have the lowest number of learning competencies in both the EM1/2 and EM3 language streams, while the most number is found in Grade 4 (EM1/2) and Grade 5 (EM3). As it turns out, the number of learning competencies expected of grade 1 pupils in the Philippines is even greater than what is expected of Singapore EM1/2 students in any grade level.

In summary, the Philippine curriculum prescribes more topics for the whole elementary level, expects more learning competencies, and repeats topics more frequently than that of Singapore.

Unique Content Areas

Table 10 shows the elementary mathematics content covered in the Philippines and not in Singapore and vice versa.

Whole Numbers

In the content area Whole Numbers, all the subtopics/lessons in the Singapore curriculum are also reflected in the Philippine curriculum. The Philippine curriculum reflects the need to emphasize the following lessons: prime and composite numbers, prime factors, Roman numbers, and order of operations involving the use of grouping symbols and exponents. Singapore students take up prime numbers and prime factorization in the secondary level under the content area Numbers and Algebra.

Rational Numbers

For Rational Numbers, all major content areas are included in the curriculum guides of both countries. In the topic multiplication and division of fractions, the Singapore curriculum deals only with the product of a proper fraction and a whole number and the division of a proper fraction by a whole number. On the other hand, the Philippine curriculum reflects more specifically that coverage should include the following topics: multiplication of fractions (product of a fraction and a mixed number and product of a mixed number and another mixed number) and division of fractions in different forms (fraction by another fraction and whole number by a fraction, etc.). Furthermore, the Philippine elementary curriculum includes basic lessons on simple probability, while in Singapore, this topic is taken up only in secondary levels 2 to 4 under the content area Statistics and Probability.

Geometry

In Geometry, all the content areas reflected in the Philippine curriculum are also present in the Singapore curriculum. However, there are enrichment topics included in the Singapore curriculum, namely, nets, 8-point compass, patterns, curves, and angles in geometric figures.

Table 10

	Content area	Mathematical content covered in t Philippines and not in Singapor	
L	Whole numbers	 Number notation Roman numbers Number theory and concepts -prime and composite numbers -prime factors Order of operations -evaluating expressions involvin 	(NO UNIQUE CONTENT)
II.	Rational numbers		(NO UNIQUE CONTENT)
	A. Fractions	 Multiplying fractions -involving mixed numbers Dividing fractions: -fraction by another fraction -whole number by a fraction -in different forms 	
	B. Simple probability	Simple predictions/chances	
III.	Geometry	(NO UNIQUE CONTENT)	 Nets 8-Point compass Patterns Curves Angles in geometric figures (as part of properties of a: square, rectangle, parallelogram, rhombus, trapezium, and a triangle
IV.	Measurement	Temperature Meter reading (for electricity and Surface area Formula derivation (for area, volu	
V.	Calculator	Parts and function of a calculator	(NOT TAKEN UP)
VI.	Statistics	(NO UNIQUE CONTENT)	1 Tables
	and a second	I was been a second	Word problems on graphs and tables
VII.	Average, Rate, and Speed	(NO UNIQUE CONTENT)	Rate Speed Z4-Hour clock Word problems on Average, Rate, and Speed
_	Ratio and proportion	 Word problems on: -inverse proportion 	(NO UNLQUE CONTENT)
VIII.		-partitive proportion	
1980	Percentage		ing price

Mathematical Content Unique to Each Country

Measurement

In the content area Measurement, the Philippine curriculum specifically reflects the inclusion of lessons on temperature, meter reading for both water and electricity to determine household consumption on a periodic basis, surface area and use of derived formulas on area, surface area, and volume. The Singapore curriculum, on the other hand, does not include these topics; instead, coverage extends up to the four fundamental operations on mass measures. Surface area is included in the curriculum for secondary levels 1 to 4 under the content area Geometry and Measurement.

Calculators

The use and functions of calculators are introduced formally in grade 3 in the Philippines and are assumed as stock knowledge in the succeeding grades. The Singapore curriculum does not reflect the introduction or use of calculators in the elementary level but does so in secondary level 1 under the content area Numbers and Algebra.

Statistics

The basic topics in Statistics are common to both countries. However, the Singapore curriculum includes more problem solving endeavors involving graphs and tables. The Philippine curriculum only includes organizing data presented in graphs and finding the averages.

Average, Rate, and Speed

Unlike the Singapore curriculum, the Philippine curriculum does not include the area Average, Rate, and Speed but does take up Average as part of Statistics (or Graphs). Grades 4 to 6 Philippine students are expected to be able to find the average of data presented in given graphs. In the Singapore curriculum, enrichment topics include speed/rate, the 24-hour clock, and problem solving.

Ratio and Proportion

In the area Ratio and Proportion, the Philippine curriculum extends coverage up to inverse proportion, while in Singapore it is only up to direct proportion. With regard to inverse proportion, the Singapore curriculum prescribes this topic only to students at the secondary levels 2 to 4.

Percentage

All lessons on Percentage in the Singapore curriculum are included in the Philippine curriculum. The additional topics as reflected in the Philippine curriculum are as follows: percent increase/decrease, commission rate, total sales, income, tax, selling price, and the simple interest-principal-rate-time connection. These subtopics, however, can be seen in Singapore's mathematics curriculum for secondary levels 2 to 4 under either of the content areas Numbers and Algebra or Integrative Contexts depending on the stream and year level.

Algebra

Algebra in the Philippine elementary curriculum is limited or even absent and the topics in the terminal year of the elementary level can reach up to pre-algebraic concepts and operations involving integers and signed numbers. The Singapore curriculum extends it up to algebraic expressions in one variable.

Summary

The mathematics curricular offerings in the elementary levels in the Philippine and Singapore schools are quite parallel. The basic difference lies in the inclusion of lessons/topics to be given emphasis either as application or enrichment.

Both the Philippine and Singapore curricula presents specific and logically sequenced topics and learning competencies within and across grade levels. Topics may vary from grade level to grade level as deemed necessary by each country's education department. Topics are organized in a spiral approach—that is, each topic is covered at appropriate levels in increasing depth (Ministry of Education of Singapore, 2006b). However, the Singapore curriculum does away with much overlap and has less number of expected learning outcomes than what is prescribed in the Philippine curriculum. These allow Singapore students more time to learn new topics and, at the same time, to take up topics more thoroughly than their Philippine counterparts.

Both countries have roughly the same number of weeks in a school year. However, in the indicated P/S Ratio, the Philippines allocates more contact time in the classroom than Singapore. This means that the time allotted in the teaching of mathematics is longer on a weekly basis in the Philippines than in Singapore.

Promotion of schoolchildren to the intermediate level in the Philippine schools is left to the discretion of the classroom teacher. Promotion of schoolchildren from the lower primary to the upper primary levels in Singapore is based on a Primary 4 streaming exercise, the results of which are used to group students by track level in Primary 5 and Primary 6. Slower Singapore mathematics students are prescribed a different curriculum where they take up topics at a lower level of study and at a slower pace.

Recommendation

Many factors contribute to Singapore's success in the global mathematics arena. Other aspects of its education system, aside from those discussed, are worth looking into to have a better idea of what made it the consistent number one in the TIMSS mathematics performance ranking. Differences in teacher qualification, quality of textbooks, and assessment, among others, can be examined. Of course, not all that has worked so well for Singapore may work well for us which may be due, in part, to differences in our economic status and population size. Having learned some of the basic curricular differences between Philippines and Singapore, it is hoped that these differences are evaluated to determine which features are worth adopting to make our mathematics education system more effective and more globally competitive.

References

Department of Education. (2007). 2002 Basic Education Curriculum.

Department of Education. (2008). Facts and figures: factsheet (as of May 12, 2008) - basic education statistics. Retrieved from http:// www.deped.gov.ph/factsandfigures/default.asp

INCA Archive. (2002). Thematic probe. Primary education: an international perspective. Retrieved from http://www.inca.org.uk/ 2280.html

- Mariñas, B. & Ditapat, M. (2000). Philippines: Curriculum development. <u>Final report</u> of the Sub-Regional Course, organized by the International Bureau of Education and the Indian Ministry of Human Resources Development. Paris, UNESCO; New Delhi, Central Board of Secondary Education, 2000. 139 p. Retrieved from http:// www.ibe.unesco.org/curriculum/Asia%20Networkpdf/ndrepph.pdf
- Ministry of Education of Singapore. (2006a). 2007 Primary mathematics syllabus. Singapore: Curriculum Planning and Development Division.
- Ministry of Education of Singapore. (2006b). Subject syllabuses-Sciences. 2001 Mathematics (Primary) syllabus. Retrieved from http:// www.moe.gov.sg/education/syllabuses/sciences/
- Ministry of Education of Singapore. (2007). Education in Singapore. Retrieved from http://www.moe.gov.sg/corporate/pdf/MOE-brochure-7dft.pdf
- Ministry of Education of Singapore. (2008). General information on secondary education. Retrieved from http://www.moe.gov.sg/ education/secondary/
- National Statistics and Coordination Board. (2008). Philippine statistical yearbook 2007. Retrieved from http://www.nscb.gov.ph/factsheet/ pdf07/FS-200705-SS2-01.asp
- SEAMEO INNOTECH. (2003). Philippines. Retrieved from http:// www.seameo-innotech.org/resources/seameo_country/educ_data/ philippines_ibe.htm
- Trends in International Mathematics and Science Study. (2007). TIMSS 1995, TIMSS 1999, and TIMSS 2003. Retrieved from http:// www.timss.org
- The World Bank Group. (2008). Singapore. Retrieved from http:// www.worldbank.org/education/globaleducationreform/ 15.LinksFromHome/15.02.StudyTour/singapore/ Singapore%20edu%20syst%20overview.htm)

Mr. Jason V. Moseros works in the Achievement/Diagnostic Test Unit of the Test Development Division of CEM. He holds the position Test Specialist IV and has been with CEM for more than five years.

Appendix A

Years in school	Normal age	Level	Curricular programme
9	25		Graduate/Postgraduate studies
8	24		
			General higher education
7	23		Prof. 1 courses
6	22		
5	21	THIRD	
4	20		
3	19		2-year post-secondary courses
2	18		Tech. 1–Voc. 1
1	17		Education
4	16		Secondary education
			General education
3	15	SECOND	
2	14		Vocational/Technical Secondary
1	13		
6	12		Elementary Education
5	11		Intermediate
4	10	FIRST	Primary
3	9		
2	8		
1	7		
	6		
	5	OPTIONAL	Kindergarten and Nursery
	4	OFTIONAL	
	3		

Structure of the Philippine Education System

Source: SEAMEO INNOTECH, 2003 (http://www.seameo-innotech.org/resources/seameo_country/educ_data/

philippines/philippines_ibe.htm)

Appendix B



Structure of the Singapore Education System

Source: The World Bank Group, 2008

(http://www1.worldbank.org/education/globaleducationreform/15.LinksFromHome/ 15.02.StudyTour/singapore/Singapore%20edu%20syst%20overview.htm)

ERROR PATTERN ANALYSIS IN MATHEMATICS: A SPRINGBOARD FOR INTERVENTION

Ma. Angeles A. Sampang Center for Educational Measurement, Inc.

Students experience many difficulties in mathematics. An examination of these difficulties can help teachers understand students better and consequently plan for remediation or intervention to facilitate students' learning of these skills and competencies. This paper examines the patterns of errors of students' responses to written mathematical tasks and questions involving the numbers strand: whole numbers, fractions, decimals, and percent (grades 1 to 6) and the algebra strand (first and second year). Error patterns are categorized as conceptual, comprehension, transformation, and technical. Error patterns are also examined according to topic: number concepts (whole numbers, fractions, decimals, percent), their operations, and their applications; algebraic expressions, linear equations, and quadratic equations. Possible approaches for intervention are also discussed.

The way students learn mathematics has been a topic of great interest to many educators (White, 2005; Swetz, 2003; Wong, 2000). Mathematics teachers know that to be able to teach well, they must know the processes students use to understand the topic they present and then adapt their teaching strategies and approaches to align with the students' mental processes. Knowledge and understanding of the students' mental processes can also assist in examining areas of difficulty encountered by them and, consequently, plan measures to strengthen and reinforce the teaching and learning on those areas.

Before the period of constructivism, teaching mathematics meant presenting a topic to the students in an oral and written manner; that is, the teacher talked most of the time using numbers and symbols to demonstrate ~ how a certain mathematical procedure is to be carried out. The teacher then asked students if the presentation was clear or if there were any questions. This "question-and-answer" part of the lesson was not a true opportunity for students to ask questions. Rather, it was a cue for the students that the teacher was through with his presentation and development of the lesson and was now ready to assign some board work or seatwork for drill and practice so students could demonstrate that they had understood the lesson. Inevitably, the students responded that the lesson was clear and understood and that there were no questions. This ubiquitous "chalk-andtalk" method has given rise to many learning problems, such as rote learning and memorization, with very little reflection on the meaning of mathematical concepts, the reason why an algorithm or procedure "works," the real-life applications of the concept and/or the algorithm, connections within " mathematics, and connections of mathematics with other disciplines.

Mathematics teaching uses what some teachers call the "CCT" approach (Class, copy this!). The students copy what the teacher has written or demonstrated on the board. This material has also been previously copied by the teacher from a reference book or textbook or teacher's manual. Thus, teaching is teacher-centered where teacher talks and shows how and students listen and repeat what the teacher has done.

These days, it is imperative that math teachers realize that one of their most important responsibilities is to "train the students to think, to combine facts, and use their knowledge and experience in making judgments and reaching conclusions" (Swetz, 2003). Thinking involves two aspects of mental activity. The first aspect is concept formation, which is the understanding of ideas about people, things or events that make them unique and different from other ideas. The second aspect is using these 'known' concepts to form new and more complex concepts. Both aspects are arrived at by the students through their personal experiences and social interactions.

In the past, teachers consider the errors made by the students as "unfortunate events that need to be eliminated and possibly avoided at all times" (White, 2005). If the error was committed orally in a class discussion, teachers tended to simply correct the error by asking another student to help and provide the correct response. If the error was committed on a written exercise, teachers corrected the error by simply marking the item red and making some notations on the paper. No attempt was made to really find what the source of the error was. Fortunately, this attitude is slowly being replaced by a more positive belief that students' errors are valuable sources of how students think mathematically. Teachers cannot really avoid students' errors. Thus, it becomes necessary to examine these errors that students make and continue to repeat. Error pattern analysis is the first step towards finding out why students make errors on mathematical tasks. It is an assessment approach to determine whether students are making consistent mistakes when answering a mathematical task that measures knowledge of concepts, computational procedures or applications, and problem-solving.

Theoretical Foundations

Newman (1983) states that when a child attempts to answer a standard, written mathematical task or question, the child undergoes a number of successive hurdles: reading (or decoding), comprehension, transformation, process skills, and encoding. Along the way, it is always possible to make a careless error. In Newman's error analysis strategy, the student is interviewed by the teacher following a specific set of questions or protocol. An error is classified as a reading error if the student is unable to read a key word or symbol in the written problem to the extent that this prevented him from proceeding along an appropriate problem-solving path. An error is classified as a comprehension error when the student is able to read all the words in the question but is unable to grasp the overall meaning of the task and is therefore unable to proceed. An error is classified as a transformation error when the student understands what the task is asking for but is not able to choose the correct operation or sequence of operations needed to solve the problem. An error is classified as a process skill error when the student identifies the correct operation or series of operations but does not know how to carry out the operations successfully. And finally, an error is classified as an encoding error when the student is able to correctly work out a solution but is not able to correctly express the solution in an acceptable written form.

Wong (2000) identifies five learning problems that are usually interrelated in errors shown in students' work:

1. They attach their own meanings to mathematical terms. Some students may be confused by the meanings of words used in the mathematics lesson. For example, a grade 1 student, when asked how much more 9 is compared to 4, may think "more" means to "add" some more and, thus, adds instead of subtracts. When asked to find the "volume" of an object, students may think of the "volume" of sound on their MP3. When told that to "eliminate" means to "get rid of" when solving simultaneous equations, they take this to mean "just ignore it."

2. They have incomplete or fuzzy meaning. Some students can only remember parts of an explanation of a concept, and so they try to complete the explanation with their own logic. For example, a teacher has just explained the meaning of $a^0 = 1$ and asked the class for the value of 2^0 . One of the students answered the value is 0 and showed the following pattern to justify his answer:

$$2^{2} = 2 \times 2 = 4$$

$$2^{1} = 2 \times 1 = 2$$

$$2^{0} = 2 \times 0 = 0$$

3. They mix up the rules. Some students mix up the rules for the different procedures they have encountered. Skemp (1976 as cited in Wong, 2000) says it is because they have no relational understanding of the steps they are doing and their long-term memory is cluttered with various rules that look pretty similar to each other. For example, a student was asked to multiply 0.4×0.6 . The student responded that the answer is 2.4. When challenged that the answer was a little high, the student explained that "when you multiply two numbers, the result is always bigger. Besides, don't you just line up the decimal point?" (Kloosterman & Gainey, 1993, as cited in Wong, 2000)

4. They focus on salient features of the concept. Sometimes students focus only on some salient features of the concept and deliberately ignore other important features of the concept. For example, many students believe that direct proportion happens when one variable increases with another variable. This is easy to remember but it is incomplete. Students do not realize that the quotient of the pair of values must be a constant or, that in a graph, the straight line must pass through the origin. In the lower grades, when asked to subtract 18 from 33 (33 - 18), a child may say, "Three minus eight, cannot be. Borrow one from three." This shows a lot of incomplete understanding of the concept of subtraction with regrouping. First, it "can be" that you subtract a bigger number from a smaller number. When they get to grade 6, the math teacher will show situations when it is possible to subtract a bigger number from a smaller number. (This is when they get introduced to integers.) Then, they will have to unlearn the "cannot be" rule. Second, we do not borrow "one from three." It is really "one ten" that we borrow from "three tens." (And some say, the term "borrow" is incorrect because when we borrow, we have to return!) That is why, now we say regroup, or decompose, or break apart a ten into ones instead of borrow.

They have a conformist attitude. Since students are trained to follow what the teacher does meticulously and faithfully (otherwise, they will not be given points), they are not used to think of alternatives and are

uncomfortable with them. They do not appreciate the reason why a procedure works, they just want to be given the rule and they will try to follow the rule as best as they could. For example, in a class, a mathematics teacher was trying to explain the reason why the rule in division of fractions "find the reciprocal of the second term and change the sign to multiplication" works. Some of the students asked the teacher for the rule or the short-cut immediately. They did not appreciate why the rule works. They reason that, anyway, the understanding of why a procedure works is not going to be assessed and scored. What is certain to be measured is their ability to apply the rule in a given division of fraction task.

Methods and Procedure

This study used the responses given by students to the Center of Educational Measurement (CEM) diagnostic test items in mathematics for the elementary and the secondary levels. Focus was on two strands: the numbers strand and the algebra strand. The numbers strand covered the following topics: number concepts; fraction concepts; decimal numbers; percent; place values; reading and writing numbers in different forms; operations on whole numbers, fractions, and decimals; renaming from one rational number form to another; and problem solving on whole numbers, fractions, decimals, and percent. The algebra strand covered algebraic expressions; operations and problem solving on algebraic expressions; first degree equations and inequalities in one variable; linear equations; systems of linear equations and inequalities; rational algebraic equations; quadratic equations; variation; integral exponents; radical expressions; and sequences.

Item analysis was done to get each item's difficulty index and discrimination index. A distracter analysis was conducted to determine the attractiveness and plausibility of the alternatives. An error analysis of the common patterns of difficulty was also conducted following these steps:

We grouped the responses of the students into three clusters: grades I to 3 as the primary level cluster, grades 4 to 6 as the intermediate level cluster and year I and II as the secondary level cluster.

Errors committed by the students for each item were classified into four error types: conceptual, comprehension, language of mathematics/ transformation, procedural/technical/algorithmic. An error was identified as a conceptual error when the student showed no understanding or incorrect understanding of a key concept in mathematics.

Examples:

- Primary level cluster. In the problem, "What is the place value of the digit 7 in 1,736?," 55% chose the correct answer— hundreds. Eighteen percent chose thousands, 15% chose tens, and 5% chose ones—these students showed lack of understanding of place values.
- Intermediate level cluster. In the problem, "Which mixed decimal has 3 in the hundredths place?," only 36% of the respondents gave the correct answer while 43% chose the digit in the hundreds place instead of the hundredths place.
- 3. Secondary level cluster. In the problem "What is the constant in the expression 3x² + 6 - 4x - 9x³?," only 48% gave the correct response of 6. Fifty-two percent gave wrong responses which showed lack of understanding of the meaning of the term constant in an expression. Twenty-five percent of the 52% chose 3 maybe because it is the first coefficient of the expression.

An error was classified as a comprehension error when the student showed misunderstanding of the context of the problem or the task.

Examples:

- Primary level cluster. In the problem, "Kenneth is 9 years old. His older brother is 14 years old. How much older is Kenneth's brother?," a substantial portion of students (22%) answered 9. The students focused on Kenneth and his age, not the brother's. This showed lack of understanding of what the problem is asking for.
- Intermediate level cluster. In the problem "For a party, Mark bought 2 ½ dozen balloons while his friend bought 2 ¾ dozen balloons. If they used 3 ½ dozen balloons to decorate the stage, what operations should be used to show how many balloons were not used?", 55% of the students gave the correct response—addition and subtraction while 45% chose only one operation or wrong operations.
- Secondary level cluster. In the item, "The solution set {x | x ≥ 7} of a linear inequality indicates that x assumes which of the following values?," only 32% of the respondents chose the correct

answer of 7 or greater than 7, while 33% chose a partially correct answer (greater than 7 and did not consider the equality sign), 24% gave an incorrect answer (the opposite of the answer 7 or less than 7) and 11% answered 7 only.

An error was classified as a language of math error when the student's error was due to a lack of understanding of the terms or symbols used in mathematics.

Examples:

- Primary level cluster. In the item that asked, "What is the odd number before 300?," only 62% of the students got the correct answer (299). Of those who chose the wrong responses, 18% gave the odd number that comes after 300, so they might have been confused with the term "before and after." Some students (12%) gave the even number, 298 that comes before 300, so the error could be attributed to the lack of understanding of what is "odd and even."
 - 2. Intermediate level cluster. In the item, "Which of the following shows fractions arranged from greatest to least?," only 52% chose the correct response. Forty-three percent chose the set of fractions arranged from least to greatest. The error might have been because they were conditioned to arrange numbers in ascending fashion from smaller to bigger values and might have forgotten that the task was to arrange from greatest to least.
 - 3. Secondary level cluster. In the problem, "The length of a box is 3 cm more than its width. If the area is 54 cm², what is the length of the box?," 42% chose the correct response (9). The remaining 58% might have been confused with the phrase "more than its width."

On the other hand, an error was classified as a transformation error when a student could not transform a sentence into a mathematical expression.

Example:

In the item, "Melissa used 1/2 m of cloth to make a dishtowel and 3/4 m of cloth to make an apron. What mathematical sentence will help you find the total length of cloth Melissa used?" Thirty-nine percent of the respondents had the wrong mathematical sentence. Instead of addition, most of the students chose multiplication to find the length of the cloth.

Lastly, an error was classified as a procedural/technical/algorithmic error when the student chose an appropriate operation but was unable to complete the operation accurately.

Examples:

- Primary level cluster. In the item, "498 92," many students (30%) had 417 as the answer. They subtracted the smaller digit from the larger digit regardless of whether the digit is in the minuend or in the subtrahend.
- Intermediate level cluster. In the problem, "What is n in 4/5=n/ 75?," 43% gave the correct response (60), while 20% might have subtracted the denominators, and 19% subtracted the denominators then added the numerator.
- 3. Secondary level cluster. In the problem,

$$2x + 3y = 11$$
$$2x - y = 3$$

What is the solution of this system of equations?," 41% chose the correct solution (x = 2.5, y = 2), while 28% simply added the x coefficients of the two equations to get 4 then substituted 4 to the second equation.

Based on the nature of the error patterns analyzed, some intervention strategies to address the difficulties and strengthen the competencies concerned will be discussed.

However, since this study was limited by the fact that there is a need to infer students' sources of errors only from their choice of alternatives, it is suggested that teachers in the field use other approaches (Newman's error analysis interview, for instance) to find why students make errors in mathematical tasks.

Results and Discussion

Error Patterns at the Primary Level

Figure 1 shows the error patterns at the primary level. To identify the error pattern, each item per grade level was examined as well as the distracter that was chosen by most of the students for the item and then the error pattern was classified. For example, if an item had 50% attainment, the other 50% were distributed among the three distracters in a ratio of 25%, 15%, 10%. The distracter with the greatest percentage which, in this case, was 25% was examined. The error found in this distracter were identified. Errors were counted according to type across grade levels. The total number of errors were divided by the total number of items for the cluster to get the percentage of error types [e.g., % of conceptual errors = (8 + 12 + 5)/78 = 32.1%].



Figure 1. Error patterns at the primary level

Across the three grade levels, 34% of the errors were of the technical/ procedural/algorithmic type, followed by the language of mathematics/ transformation type (29%), then the conceptual error type (25%), and comprehension error type (12%). Error patterns of the technical/procedural/algorithmic nature are frequently found in the responses of the primary students. In fact, 34% of the errors were of this nature. Common errors found with this type involve *addition and subtraction with regrouping, and zero difficulty in subtraction.* Many students have difficulty in addition with regrouping. When asked to add 45 and 37, a common error answer is 712. The students forget to regroup 12 ones into 1 ten and 2 ones so they can "carry over" the 1 ten with the other tens. They simply write 12 and proceed to add 4 and 3. Subtraction with regrouping is more problematic. Aside from their difficulty in the process of regrouping, students tend to forget that they have regrouped or decomposed a ten into ones and, thus, the number of tens is now 1 ten less. Other students may have "mixed up the rules" when subtracting with regrouping in the tens and the hundreds place. The problem is increased when regrouping involved zeroes in the ones and the tens place.

Error patterns as shown are also driven by difficulty with language. The students in this level experience difficulty in reading, writing, and speaking. Since the students are just at the start of their learning experiences, the language of mathematics comprises of vocabulary words that they might find difficult especially if some of these words are used rarely outside of the mathematics classroom. English words that may have a different meaning in mathematics add to the language difficulty of students. English words, such as *first, second, before, after, between, more, less, every, both, equal, left, right, greatest,* and *least,* may be difficult for some students. Students may be unfamiliar with mathematical terms such as *sun, remainder, difference, factor, product, commutative, order, grouping, associative, numerator,* and *denominator.* They may not understand a fraction decoded into words such as *two halves.* They may think of the common meaning of a term instead of the mathematical meaning (e.g., odd as in unusual instead of not exactly divisible by two).

Many error patterns are also found to be conceptual in nature. This means students at the primary level do not show complete understanding of basic ideas in mathematics such as *place values of ones, tens,* and *hundreds,* and *concepts of addition, subtraction, multiplication,* and *division.* Students at the primary level can read and write numbers up to the thousands, but their concept of *grouping by tens and/or regrouping or decomposing tens into ones, hundreds into tens* is at best incomplete. Students at the primary level can do their basic addition, subtraction, multiplication, and division facts, but their understanding of *the meanings*

of these operations are incomplete. They show some difficulty in determining which operation to use in given real-life situations. Many of them do not see that subtraction is the opposite of addition. Thus, a task that involves a missing addend is difficult. When asked " $3 + __= 7$," many of them give the answer 10 because they see the plus sign and just think of addition, so they add the two numbers given and come up with the sum of 10 instead of the missing addend of 4. The same difficulty is also present with the relationship between multiplication and division. Many of the primary level students are not aware of the fact that they are opposite operations of each other. They do not see that three 2's = 6 is similar to asking how many 3 there are in 6; or that 3 x 2 = 6 is the same as 3x = 6.

Error Patterns at the Intermediate Level

In the intermediate level, 51% are technical/procedural/algorithmic errors, followed by conceptual errors (28%), then language of mathematics/transformation errors (11%), and comprehension errors (10%).



Figure 2. Error patterns at the intermediate level

More than half of the errors observed in the intermediate level are technical/procedural in nature. This maybe because students have mixed up the *rules involved in the operations of whole numbers, decimals, and fractions.* For example, when asked to add "1/2 + 1/3", a common error answer is 2/5 because the two numerators and the two denominators were simply added. When asked to find the product of "4.60 x 0.12," a common error answer is 5.52 because, after the calculation is done, the student just brings down the decimal point. In addition to these rules in the operations are the peripheral tasks of *renaming from fraction to decimal, and to percent and vice versa; finding the greatest common factor to reduce fractional answers to their simplest form; finding the least common multiple to build fractions so that the fractions will be similar; renaming improper fractions to mixed numbers and vice versa.*

Error patterns that are conceptual in nature are also identified in the intermediate level. The conceptual errors in this level are due mostly to lack of *recognition of situations for which multiplication and/or division* is appropriate; properties of the operations; equivalent fractions; understanding and using symbols such as the fraction bar and the decimal point; other rational forms for fractions; rounding, comparing, and ordering fractions; visualizing fractions that are close to 0, 1/2, or 1, decimal place values; ratio and proportion; percent; and identifying the base, rate, and percentage of a given problem.

Errors involving language of math and comprehension are almost the same in percentage. An examination of the items shows that the errors are due to the fact that students have to deal with terms such as *numerator*, *denominator*, *proper fractions*, *improper fractions*, *like fractions*, *unlike fractions*, *tenths against tens*, *and hundredths against hundreds*. The decimal place values of *tenths*, *hundredths*, and the like are specially difficult for students because they sound like *tens and hundreds* which students are familiar with since the primary level. These new terms are also spelled somewhat similarly. Besides, they are not heard in life outside of the classroom. It is only in the classroom where the students are asked to read 2.3 as two and three tenths. Outside of the mathematics classroom, *2.3* is read as *two point three* and is accepted and understood. Thus, when a task asks students to identify the place value of a digit in the *tenths*, *or thousandths*, *or millionths*, students do not take notice of the *ths* but simply think of *tens*, *thousands*, and *millions*.

Error Patterns at the Secondary Level

In the secondary level, 45% are technical/procedural/algorithmic errors, followed by conceptual errors (30%), language of math/transformation errors (14%), and comprehension errors (11%).

One of the most common errors of students that is technical and procedural in nature is in the use of the negative and positive signs before an expression. For example, in a task that asked, "If 3x - 20 = -10 - 2x, what is the value of x?," only 37% of the examinees got the correct answer. Thirty-two percent of the examinees chose 10 as their answer because -2x was still negative when transferred to the left side of the equation. In other cases, some students seem to just ignore the directions of the task. When asked to simplify an expression, they seem to ignore the word *simplify*. When given inequalities, some students seem to ignore the equality sign and simply focus on the inequality signs.



Figure 3. Error Patterns at the Secondary Level

Most of the conceptual errors may be due to a lack of understanding of the concept or confusion about the mathematical meaning of terms, for example, constant, coefficient, binomial, exponent, abscissa, ordinate, variable, coordinates, rational expression, direct square variation, inverse variation, radical expressions, and other terms in this level.

Some of the errors that are identified as language of mathematics or transformation error may also be due to lack of understanding or confusion about terminologies. For example, in the item "Which inequality means *four more than a number is at least 39?*," only 21% of the students got the correct response. Many of the students chose ≤ 39 instead of ≥ 39 because of the phrase *at least*. Other students translated *four more than a number* as 4x.

Error Patterns Across Clusters

An analysis of variance on the types of errors across clusters was conducted. Figure 4 shows the summary of error patterns for the three clusters. As noted in Figure 4, 43% of the student errors are technical/ procedural in nature, followed by conceptual errors (28%), language of mathematics/transformation (18%), and comprehension errors (11%).



Figure 4. Summary of error patterns

Table 4 Summary Table ANOVA

	Sum of squares	Df	Mean square	F	Sig.
Between groups	1754.916	3	584.972	12.037	.002
Within groups	388.793	8	48.599		
Total	2143.709	11			

Additional analysis of variance on the types of errors across clusters was conducted. Results presented in Table 4 show that there are significantly more technical errors than comprehension errors (mean difference = 32.2) and more technical errors than language of math/transformation errors (mean difference = 25.37). This means that the students understand what the task is asking them to do but makes an error, nevertheless, because of carelessness in computation and confusion in applying the steps of an algorithm.

Conclusion

Results of the error analysis show that, in general, students across levels have as much difficulty in conceptual understanding, as in computations and applications. They have more technical and procedural errors compared with conceptual, comprehension, and transformation errors.

Implications For Teaching

In the Primary Level

Learning theorists, like Piaget, Bruner, and Gagne, are all one in stating that mathematics teaching, especially in the early grades, should be based on concrete experiences with children manipulating objects and interacting with mathematical principles and concepts (Swetz, 2003). Concrete-based instruction is, thus, desirable. From concrete experiences, one can move to semiconcrete or pictorial representations and, finally, to the abstract or symbolic representation of the concept that is under study. Thus, when teaching the concepts of addition and subtraction, the teacher should begin by making the children put things together, take away parts of a set or compare two sets. Children should be asked to verbalize their experiences (e.g., two and three are five, six take away four are two). They can be asked to examine pictures that show situations calling for addition or subtraction. Spoken sentences are replaced with symbols: 2 + 3 = 5. When teaching the basic facts, relational thinking should be emphasized (e.g., if 5 +5 = 10, then 5 + 6 should be one more than 10; if 3 + 4 = 7, then 4 + 3 also = 7; if 3 + 3 is the same as 2×3 or 3 doubles = 6, then 2×4 is just 4 doubles = 8. If 4 is 2 doubles, then to solve for 4×3 , just double 3 = 6, then double 6=12). When teaching addition and subtraction with regrouping, students should have concrete experiences in putting together ones to make a ten and breaking apart a ten into ones. Because children at the primary level are just beginning to learn mathematics concepts and terms, there is need to teach meanings of terms explicitly (e.g., Emphasize that equals means "to balance" or "is the same as"). To help primary levels become more proficient in problem solving, different problems should be given to solve. A few problems that show different situations is more helpful than many problems of the same situation, thus, exposing the students to different situations that call for addition, subtraction, multiplication, and division. Models can be used to show the different situations that require different operations. For example, in teaching the concept of subtraction, most of the time, only one concept of subtraction is used, as a taking-away process forgetting to show the students that subtraction also means comparing. In a test, students are asked to find how many more ribbons Ana has than Ruby if Ruby has 6 ribbons and Ana has 9 ribbons. This calls for subtraction, but the students have not been introduced to similar situations and are therefore at a loss on how to proceed. Another situation where subtraction is appropriate is in equalizing. For example, "TJ wants to save P15 to buy some marbles. He has saved P7. How much more does he need to save?" This situation calls for subtraction, but it is neither a takingaway situation nor a comparing situation. It is an equalizing situation. When the topic is division, teachers need to present students with situations that call for measurement division, the number of two's in 8 and partitive division, e.g., the number of times 8 can be divided by 2. Thus, there is a need to present the students with different situations that call for the appropriate operations.

In the primary level, since the students find all three aspects as easy or as difficult, there is a need to balance concept formation, computations, and applications. Plans for learning experiences should take into consideration mathematical understanding of the basic concepts, insights, and connections of known concepts and facts to build on other concepts and facts to progress in mathematical skills.

In the Intermediate Level

Just like the students of the primary level, mathematics teaching in the intermediate level should begin in the real world with concrete experiences that are within their developmental level. They also learn better through doing and discussing what they are doing. For fractions, different models can be used to help students understand basic fraction concepts: region model using circles, squares, rectangles, and triangles cut into equal parts; set model using counters such as buttons, beans, shells, and popsicle sticks; and distance model using the number line. Basic fraction concepts include meaning of fractions such as equal parts of a whole, as ratio, and as division (fraction bar means division); equivalence of fractions so they will have experiences that will show 1/2 = 2/4; fractions that are close to 1 or greater than and equal to 1 so they can visualize proper, improper, and mixed numbers. Similarly, conceptual understanding of decimals can start with concrete models: region model such as a square divided into 10 parts, into 100 equal parts, or into 1,000 equal parts; a distance model such as a meterstick, the decimeter as tenth, the centimeter as hundredth, and the millimeter as thousandth; a value model using money such that # 1,000 is one whole, # 100 is one tenth, # 10 is one hundredth and # 1 is one thousandth. Operations with fractions and decimals can also be learned better if teachers begin with concrete experiences then move to pictorial and then to symbolic. The learning of the operations can also be facilitated by making connections and finding patterns and relationships using properties of the operations.

In the Secondary Level

A program of intervention in algebra should include a balance of conceptual understanding, computational and procedural competencies, and problem solving. Special attention should be given to the academic medium of instruction and the mathematical meaning of new terms and multiple meaning words specially, since to many of these students, English is a second language (or even a foreign language). Algebraic concepts and terms, such as variable, power, functions, relations, sequences, expressions, and so forth, should be taught explicitly using varied examples and situations that define the concept. The examples and situations must be within the real and curricular experiences and interests of secondary level students (who are teenagers). They must be presented in a 'ogical

sequence from the simple to the complicated, from the more concrete to the abstract. Finally, the students can be provided with tasks that will encourage them to explore and investigate so they can show their mathematical understanding and problem solving strategies. Such tasks will also help identify sources of students' errors in understanding, computations, or applications.

References

- Heddens, J., & Speer, W. (2006). Today's Mathematics concepts, and classroom methods, and instructional activities. MA: John Wiley & Sons, Inc.
- Newman, M.A. (1983). Strategies for diagnosis and remediation. Sydney, Australia: Harcourt Brace Jovanovich.
- Swetz, Frank J. (2003). Teaching Mathematics to children. Ateneo de Manila University: Office of Research and Publications.
- Tucker, B.F., Singleton, A.H., & Weaver, T.L. (2006). Teaching Mathematics to all children: Designing and adapting instruction to meet the needs of diverse learners. OH: Pearson Prentice Hall.
- White, Allan L. (2005, December). Active Mathematics in classrooms: Finding out why children make mistakes – and then doing something to help them. Square One, 15, 15-16.
- Wong, Khoon Yoong. (2000). Enhancing Students' Learning Through Error Analysis. Department of Science and Mathematics Education: Universiti Brunei Darussalam.

Ms. Ma. Angeles A. Sampang works in the Achievement/Diagnostic Tests Unit of the CEM Test Development Division, which currently maintains over 50 diagnostic tests in the four core subject areas of English, Mathematics, Science, and Filipino. She is currently the Unit Head and has been in the service for over 27 years. She holds a Master of Science degree in Educational Measurement and Evaluation which she acquired from the De La Salle University-Manila.

PANEL REACTIONS

Dr. Rosemarievic Villena-Diaz on Mr. Jason Moseros' Paper

I would like to congratulate the author for coming up with a very meaningful paper and commend CEM as well for supporting an endeavor as such. I understand, CEM is exerting a lot of effort in raising the quality of education in our country by producing quality instructional materials and by updating and upgrading teachers through its Professional Education Program Series (PEPS).

I would certainly agree with the author on choosing the Singapore Mathematics Curriculum as his benchmark of study. After the release of the Trends in International Mathematics and Science Study (TIMSS), everyone would like to examine what is going on within the Singaporean mathematics education.

Highlights

The comparison of the curriculum in terms of design, content, topic exposure is clearly stated in the paper.

TT: . 7.7: . 7	10.00	2000	the second second	la cora	A
Highlight	1.	UM	curricui	um	aesign

Singapore
Education system is centralized.
 Guidelines and suggestions on the methods of teaching are highlighted, together with a clear statement of the intended standards of achievement.

The Philippines is a diverse country. Such diversity, expressed in the different dialects, cultural and ethnic variety, economic conditions, or geography should be considered in implementing a curriculum. Indeed, it is but proper for the teacher to decide on the teaching/learning activities appropriate for his students. But, how versatile are our classroom teachers in implementing the curriculum?

Since the system is decentralized, in the end, who would be deemed accountable for the poor performance of our students?

Are the Singaporean students homogenous that their lesson plan works for all types of learners? How do they train their teachers so that these teachers seriously implement their prescribed plans?

Is it possible for Filipino teachers to use the same set of textbooks, implementing the same set of lesson plans, and administering parallel test questions in a certain mathematics topic? Of course, many Filipino teachers would welcome the idea of not writing the plan, and just execute one which is close at hand or is readily available.

What if a teacher feels that his skill is inadequate to implement a certain lesson plan? What options does he have? Will he be spending more than half of the period checking attendance and executing games for his motivation? Or, will he be allowed to design his plan appropriate for his skill without considering what is best for the students?

Highlight 2:	Promotion of schoolchildren from primary (Grade 1-4)
	to intermediate level (Grade 5-6)

Philippines	Singapore
 Promotion is left to the discretion of the classroom teacher. 	 Promotion is based on the child's performance in the "Primary 4 Streaming Exercise"

Can we adapt this system in the Philippines? Will it be different from the National Achievement Test?

How will the English and Natural Science advocates accept this?

Will it be the same as the "Bridge Program" which was implemented some years ago?

Philippines	Singapore
 average time per week for Math is 350 minutes 	 average time per week for Math is 285 – 306 minutes
 covers a total of 69 topics 	 covers 60 topics in EM1/2 covers 52 topics in EM 3
 covers 25.8 topics per grade level 	 19.7 topics per grade level in EM 1/2 21 topics per grade level in EM3
 spends 2.1 grade levels per topic 	 1.9 grade levels per topic (EM1/2) 2.4 grade levels per topic (EM 3)

Highlight 3: The Mathematics Curriculum

Both countries' curricular offerings are quite parallel, but just by looking at the data, we can say that the Philippine Math Curriculum offers more than the Singaporean curriculum in terms of length of time and number of topics.

But why is their students' achievement in terms of Mathematics higher than ours, as reflected in the results of the international examinations?

Are we overloading our students? Is there such thing as "overloading"?

Consider the case of our science high schools whose curriculum requires their students to study more topics and to have longer contact hours in school.

The results of the 2003 TIMSS reveals that the second year high school students from three science and technology oriented high schools performed better in mathematics compared to other Philippine sample students from public and private high schools. Their scores are significantly higher than those of Australia, USA, India, Malaysia and Russia.

Will we consider that overloading is the issue or should we examine further what are the realities in the field?

Do we cover all the topics included in the designed curriculum?

How well is the contact hours spent by the students and by the teachers?

Panel Reaction: Dr. Rosemarievic Villena-Diaz on Mr. Moseros' Paper

68
Philippines	Singapore	
 Curricular review and reform takes place whenever DepED sees a need to do it. 	 Holds curriculum planning and review on a regular basis (8 -10 years, then 6-yr cycle) 	
 DepED assures the continuous review and refinement of the curriculum to ensure that it responds to changing needs and demands. 	 There exists a CURRICULUM FRAMEWORK 	

Highlight 4: Curricular Review and Reform

Having revisited these highlights, one would say that the Singaporeans are well guided by their framework, a framework which seems to be missing in the Philippine document.

I agree with the author's recommendations, one of which is to examine differences in teacher qualification, quality of textbooks and assessment, among others.

The challenge, however, is to understand the reasons for these differing performances and to recognize effective policies, program of actions and best practices. More importantly, we need to reflect on the culture prevailing with regard to the role and importance of mathematics. How much do our Filipino students value mathematics? What values and ideals are imparted during classroom discussions? Are these values and ideals shared at home and in the community? What level of mathematics competence should an individual manifest/demonstrate to qualify him to grade 5? to high school?

Dr. Rosemarievic Villena-Diaz is an educator with 17 years of teaching experience both in the undergraduate and graduate teaching programs of the Philippine Normal University, a teacher-training state university. Over the period, she has been a lecturer in preservice and inservice training programs. Her training and experience include designing, reviewing, and revising curricula and preparing instructional materials across levels. She has also served the graduate school in various capacities: as adviser of graduate students in thesis writing and as examiner in graduate theses oral examinations. She received her Bachelor of Science degree in Mathematics from the then Philippine Normal College, Manila, and her Master of Science in Math and her Doctor of Philosophy in Science Education major in Math from the De La Salle University.

Dr. Lynda Parreño on Ms. Ma. Angeles Sampang's Paper

The paper presented by Ms. Sampang identified the most common error patterns in Mathematics in the following clusters: Primary level cluster (Grades 1 to 3), Intermediate level cluster (Grades 4 to 6), and Secondary level cluster (first and second year high school).

These error patterns include: (1) Language of Mathematics and Transformation errors; (2) Comprehension error; (3) Conceptual errors, and (4) Technical, Procedural, and Algorithmic errors.

I would like to elaborate further on the paper presented by Ms. Sampang specifically on the possible strategies that can be employed to improve Filipino students' learning of mathematical concepts and procedures.

I believe that there is a need for comprehensive mathematics curricula in schools, which will build on and enrich previous topics as students progress through the educational system. Particularly, a continuous mathematics curriculum from N to Grade 6 or N to 4th year high school must be planned in accordance with the general curriculum set by the Department of Education (DepED). Thus, the utility of providing mathematics teachers with instructional frameworks organized by and coordinated across cluster levels to guide their teaching and the importance of remedial lessons after each topic must be underscored.

Given the results of the study, the teaching strategies or approaches implemented by the teachers concerned might not have been aligned with the students' mental processes and their current level of skill with the topic. Exercises should effectively motivate students to relate their mathematical knowledge/skills to real-life applications as well as to the concepts in other disciplines, and school administrators should carefully select textbooks/ workbooks to be used to ensure the achievement of the school's set objectives for mathematics education.

Dr. Lynda Parreño is an educator with a background in mathematics and special education. Dr. Parreño received her Bachelor of Science degree from the FEATI University, both her MA in Educational Management and MA in Special Education from the Philippine Normal University, and her Doctor in Education in Educational Management from the Philippine College in Health Sciences, Manila. She started as a Mathematics teacher and became an educator at the college level in various schools in Biñan, Laguna. She has recently served as High School Special Educator at Patterson High School in Baltimore, Maryland, USA.

OPEN FORUM HIGHLIGHTS

Morning Session

Dr. Gureng: Good morning. The problems posted by Dr. Yeap kept us awake the whole morning. The whole message of Dr. Yeap Ban Har serves as a wake up call for curriculum experts and us mathematics educators, including the DepED. To be sure that we are still awake, I invite you to please participate in the open forum. Dr. Yeap will be very happy to answer all your questions ranging from basic skills to basic applications to problem solving. The floor is now open. Please identify yourself, your school, your age or specialization like elementary or high school, or if you are an administrator, or anything with which you can be identified. I will read the first question: How do we construct problem solving questions like the one in the example. And who wrote those items? Is it Dr. Yeap, or the national government?

Dr. Yeap: Our national examination, and these are the ones you saw, is handled by the local examination board. It's called the Singapore Examination and Assessment Board (SEAB). So SEAB is responsible for the construction and the running of the entire examination. So the simple answer to your question is, who constructs these items? It is the examination board. Who are the actual people constructing the items? It's a national secret. If you are invited to write items, you cannot publicly announce like just now that you write items for the SEAB. It's a national secret. Not even your spouse can know that. You will write the items, you will stop working, and you will never know when it will appear and in what form it will appear. There is a structure to ensure the integrity of the examination. To put it very simply, if the item is good, it can come from school teachers, department of Mathematics, Mathematics educators, people like me who train Mathematics teachers or Mathematicians, teachers in the university or any other person that are appointed as item writers.

Dr. Gureng: I had a chance to observe a Math class in Singapore. And the person opts not to identify the school. The teachers in the classroom are giving materials that are already printed. And by interview, topics from textbooks are authorized, or sanctioned, by the government. In other words, some teachers do not give additional items. So no teacher-made test for the class. Is that true? Meaning the teachers do not add some more items to those things that are printed.

Dr. Yeap: All right. In Singapore, we use a textbook. There are textbooks that you can use that are approved by the Ministry of Education. We have a textbook accompanied by what we call a workbook. So that alone is enough to satisfy the requirement of the curriculum. Technically, the teacher will do that. But very, very few teachers will do just that. Because in a textbook, although there are problem solving, there will never be an element of newness. So it is more common practice in most schools that their Mathematics department will have a collection of worksheets. And these are not from the textbook. The textbook materials are in a book form. If what you see is in a book form, it is a standard workbook and textbook. If what you see is in the form of handouts, sheets of paper, that means these are additional materials in the form of worksheets. These are collated by the Mathematics department. Although some teachers may just use the workbook and the textbook, and technically those are fine, very few schools actually do that because they will never prepare the students for the new items. So depending on what you see, you can come to the conclusion, if what you see are sheets of paper, that means those are additional materials. Standard materials are always in a form of a booklet.

Dr. Gureng: The next question: What is the typical class size in a Singapore classroom and how many minutes or hours are devoted for Math class per week?

Dr. Yeap: The maximum class size for secondary school is 42, in elementary school it's 40. The first and second grade levels were recently allowed to have 30 pupils. So grade 1 and grade 2 have 30 pupils. The rest of the elementary grade has 40. The secondary has up to a maximum of 42 pupils. How many hours do we spend on Mathematics? Typically, an hour a day in the elementary grades. The younger children get more because we don't teach Science. So the time saved for Science is distributed between Mathematics and English. A range of 5 hours a week. The younger students get a bit more and the weaker students in the 5th and 6th grades get a little bit more. In secondary school, the time is a little bit less than 5 hours a week, it could be four and a half.

Dr. Gureng: The next question: We bought Singapore Math books from a local supplier but we were not given a copy of the Mathematics curriculum. And the reason of the supplier was, they got stopped at the Customs. I don't know how is this. How can we have a copy of the entire Math Curriculum of Singapore schools?

Dr. Yeap: The Singapore school system is from Grade 1 to Grade 6, elementary, and Grade 7 to Grade 10, secondary. Grade 11 and 12, junior college. So it's 1 to 12. The curriculum is publicly available, you can visit the Ministry of Education website. Just google Ministry of Education Singapore, go to Ministry of Education website. All curriculum, with every single subject including Mathematics of all levels are freely available. So you can just go and download it. It's there.

Dr. Gureng: Usually fraction is one of the weaknesses of our elementary students. How do you strengthen it? Any methodology that you can suggest?

Dr. Yeap: Fractions is actually a difficulty for all students all over the world. And we can understand why. Earlier I showed you a problem involving fractions, the one-quarter and one-fifth. And you noticed how I used diagrams. In Singapore, we refer to this as the model method. It deserves the name we refer to it. The fact that it has a name means it's very commonly used. It's all over in the textbooks in Singapore. It's a very common method that children use to solve problems. And it's actually very useful for problems involving fractions, ratios, percentage and proportion. And you notice in the computation I showed you just now, although this is a fraction problem, none of the calculations involve fractions. The complexity of fractions is now embodied in the diagram. That is one way we help our students understand fractions and, actually, many other concepts, easily. There is a strong emphasis on the use of visuals in the textbooks. So one simple answer to you is, how do you help students view the fractions? It's through the use of diagrams such as this one.

Dr. Gureng: The next question is about whether the curriculum in Singapore is really too crowded or not. In the first and second grade, you told us that you sacrificed the Science subject to give way probably to some important subject, or core subjects, like you mentioned. May we know the subjects that you have in Grade 1 & Grade 2, probably just to check whether it's really too crowded or not.

Dr. Yeap: In Singapore, the core subjects in elementary school are English and Mathematics. So the emphasis is really on English and Mathematics. We have other subjects, of course. There's Physical Education, Art, Music, and there's the whole language. There's a whole slew of subjects: Ethics and Moral Education, Health Education. But the emphasis is really on English and Mathematics. Although international comparison, like the

Open Forum Highlights

one you will hear later today, indicates that our curriculum is not too heavy, we feel that it is too heavy. In the year 2004, the Prime Minister, in a national speech, urged the teachers to do something that will sound a little bit strange to you. The slogan that is used is: *teach less, learn more*. The prime minister urged schools to think of ways to teach less. For example, in Mathematics, please don't teach them the formula to find the area of trapezoid, triangle, rectangle, rhombus, or whatever else you need to. Just teach them the concept of area. If you know how to find the area of rectangle, that's good enough. Then lead them to figure out how to use the area of rectangle to find the area of an object. How do you construct a trapezoid from a rectangle? So in our opinion, it is still too crowded in the sense that our teachers are able to cheat. We want our teachers to focus on the basic and then they can figure out the rest of it.

Dr. Gureng: I'll delay the reading of the next questions to give way to some of you who might prefer to ask questions directly from the microphone in the center aisle.

Guest: Good morning, Dr. Yeap. I am Shiela Yao from MGC New Life, one of its administrators. Our school is one of the schools that got too excited with the Singapore Math and jumped into it without studying it. And at this point, the teachers, I'm with two of them now, are having a hard time teaching the subject simply because we were not taught the pedagogy of teaching unlike what you have in Singapore. What I want to know is, do you have trainings available locally? Because I'm thinking it would be too expensive to send representatives to Singapore.

Dr. Gureng: Excuse me, Dr. Yeap, there's a similar question to that. I'm going to post it so you get to answer two questions. What short term Math training seminars are there to which we can send our teachers to get enough Math attitudes and skills much like Singapore's National Exam. And how do we gain access to these seminars/trainings?

Dr. Yeap: Number one, I'm sure there are many similar high-quality professional development programs in the Philippines already. I'm sure that there are some universities or agencies based in the Philippines which are already conducting such high-quality seminars. The second part of the question that you are asking is "if there are Singapore trainers from Singapore institutions who come here to train teachers?" The answer is yes. I'll give you some example. The distributor who distributes the Singapore textbooks, runs 5 to 6 seminars a year. So I suppose you may

get information from them when these seminars happen and they bring in Singapore trainers. Sometimes I come. I got colleagues from English, Language, and Science who also come. Other Mathematics colleagues also come and we conduct trainings. Obviously, the training is more intensive and more of an overview rather than going into specific details, which we do with our free service just in Singapore. Secondly, there are some schools in the Philippines who take the initiative to organize such sessions as well. For example, in May, I and about 10 of my colleagues came to Manila and conducted summer workshops in the Ateneo Grade School for teachers from all levels, grade school all the way to high school. And obviously, the teachers from the Ateneo school, came. But I believe there are 7 or 8 teachers from other schools. So I suppose if you are connected to your network, find out when these things happen.

Dr. Gureng: How is the situation in a Math classroom? Is it much of chalk and board work? Or lecture, or seatwork exercises where the students are engaged in this endeavor?

Dr. Yeap: I think there's a mixture of both. There is a time for hands-on activities in the desk, time for discussion, and also a time when the teacher will be doing some explanation. Our teachers are trained to mix strategies, and based on my observation of teachers in school, especially in the elementary school, they are applying a lot of them. I actually have a video of a classroom in Singapore and, if there is time, I will show it to you. That will probably answer the questions best. Having said that, I don't think we can generalize what happens in a classroom. What I'm telling you is more of what the system is expecting the teachers to do. The teachers are expected to engage the students in the use of concrete materials, discussion, as well as teacher presentation. But you will never see an elementary school teacher giving a lecture to a class of 8 year olds. I don't think it happens anywhere, or should happen anywhere.

Dr. Gureng: I'll get to some questions. I put them together. The first question is about the students of Singapore having no fear at all or how are they motivated? Is it self-motivation or motivation from the teacher?

Dr. Yeap: I think it is probably not true to say that students find Mathematics totally easy, or find those problems easy for sure. That's not possible. They may find it challenging. There are some students who are not performing well, I would say 10% of the students. But I will not go to the extent and say that a lot of students fear Mathematics. They may find it

Open Forum Highlights

challenging. Sometimes they fail, sometimes they succeed. I think that's a situation we have. Our children are not different from your children, they are the same. They will struggle a bit. And they will succeed sometimes. How do we motivate our students? Our curriculum emphasizes the use of visualization. Our system emphasizes on the use of human qualities, which we all possess, to learn Mathematics. So because we are trained to use our strength, we will succeed. Except the 10%, or 10% will not work. Rather than focusing on memorization and tedious procedure. Again, the best way to answer that question is to show you a short video on how we encourage our teachers to motivate Grade 1 children to learn through the use of various methods.

Dr. Decenteceo: I understand that you don't have a licensure examination for teachers. So how do you qualify them?

Dr. Yeap: In Singapore, if you want to teach in a school, which are all public schools, you have to get training. In my case, it's my Institute because it's the only one. So you need to graduate with a Diploma in Education from our institute to become a teacher. So that is the entry point. Once you graduate with a Diploma in Education, you can teach in a school. Every year, there is an evaluation of teachers. It's not a test. It is based on the proper teaching as well as non-academic work or extracurricular activity like dance club or swimming as well as maybe the organization of one or two major events in the school like speech day or sports day, or some other special events. Generally, the teachers are evaluated both in teaching competency as well as in the administrative competency. At the end of each year, they are given an evaluation report, which is given by their immediate supervisor, in consultation with a panel. The immediate supervisor could be the principal, head of the department or a senior teacher. This person, in consultation with a panel that is from the school, will evaluate every single teacher in the school. The teachers are given grades from A to C, which are all okay. A grade of D is not okay. If you receive a D, you are first given counseling. And they will say to you, "You know you are not achieving the goals of education, of the curriculum. You are supposed to engage the pupils but you are talking too much in class and the students seem not to be interested in what is happening. You go and lecture your 2nd grade, but they are not learning. Perhaps you want to try some of these strategies. Perhaps you can sit in So and So's class. And maybe discuss with your other colleagues. And in six months time, we sit out again to see your progress." So we give teachers counseling this way. We have a very kind system, we rarely sack teachers.

Open Forum Highlights

Dr. Gureng: Those things that you've identified as basic skills, they are advanced levels in other places. So I agree with you. You showed us samples of novel problems or items of complexity. Are those from the grade school? Or primary school?

Dr. Yeap: Every single item I've shown you is from the grade 6 national examination.

Dr. Gureng: It doesn't teach you how to make novel or item complexity in high school, or in your case, secondary level.

Dr. Yeap: I will now show you one example from a Grade 10 national test. It's based on a chapter on vectors. The problem is this: A fly, F, starts at a point, with this position vector i + 12j cm. So basically, vectors will help you locate positions, like on a graph paper. So position vector, i + 12j that is on a ground, x = y, y = 12. That's what it means. So a fly's starting point and the position vector is given. It crawls across the surface with a certain velocity. The velocity is given at 3i+ 2i centimeter per second. You know what that means? It simply means that it is moving in that direction. For every three-centimeter it moves forward, it is also moving upward two centimeters. So that is what a velocity of 3i + 2j means. Every time the x position changes by 3, the v position changes by 2 every second. So the distance and direction of the fly is given. At an instance that the fly starts crawling, the spider, appropriately called S, is at the point with this position vector. The position vector of the spider is given, which is 85i + 5j, and sets off across the surface with a velocity. This velocity is given at -5i + kj. So the unknown, K. So you do not know that when the spider moves backward by 5. What is the change in the v-value? So that is unknown, the velocity of the spider. Given that the spider catches the fly, calculate the value of k. That is an example of a problem that is novel. This is not found in a textbook. They have never solved this problem before in a textbook. But the student will then think "Oh, if the spider catches the fly that means they must be in the same place, at the same time. So I need to know where is the fly after a certain time. That is at time k. Because I don't know when they are going to meet." Let us make that unknown. At time k, where is the fly? Since the fly is at y = 12, x = y, y = 12, and every second that x changes by 3 after a time k, we will change by 3k. So eventually the fly will be at y + 3k. One is at the original position and every second it changes by 3. So in three seconds it changes by 3k. Similarly, we know the yposition for the fly is 12. In other words, we know the time speed, where the fly is. By the same procedure, the students can also find the final

Open Forum Highlights

77

position of the spider. Since they are at the same place, so the two values must therefore be equal. At the same time, the students will also then solve this. I will not show you the complexities of the solution, but you will see that in this case, the student has to apply what he learned in vectors in a situation he has never encountered before. However, the problem with our Grades 10 and 12 examinations is that these kinds of problems are far and between when the examination was set by the Cambridge examination certificate. These were the ones that the local examiners closed in after receiving the papers from the Cambridge examination curriculum. But starting this year, everything is from the local examination syndicate and therefore these kinds of problems will now be more common in our secondary school level. So although the primary school national test comprises of a lot of these kinds of tests, in the secondary school these don't exist. It's too uncommon to have the teacher make problem solving the focus of the examination. So I showed you one example from the Grade 10 test to see what we meant by problem solving.

Dr. Gureng: The question is, how do you balance the mastery of computational skills and critical thinking? We have drills in our Math class with room for creativity and learning Mathematics in our classes.

Dr. Yeap: In our system, we do not see the dichotomy between basic skills and creativity, although the first four grades focus more on basic concept development, and the fifth grades onwards focus more on problem solving. I give you now one example what I meant, I don't see the dichotomy between the two. For example, addition. When the children are taught addition, let's say 39 + 17. In the first grade, they are taught one method. Let me show you one. Adding tens and adding ones. In the second grade, they will learn bigger number addition including the standard method. But, there is one chapter on mental calculation, wherein they will see again the addition of 39 + 27. And in that chapter, the main idea is how can you add 19 and 27? Sure, you already know that last year and in the ones, and in the tens, you did some renaming and carry. But how else can you do it? It's what the textbook and students would think. And some of the methods were discussed in the textbook and include 19 + 27. Can you imagine moving one from 27 to 19? Making the 19, 20 and consequently, the 27 becomes 26. Oh, so another way to add 19 & 27 is to add 20 + 26, which we have already done. Another student will raise his hand and say, "Oh, I have another method, 19 + 27, why don't you do 20 + 27 first because it's so easy. Afterwards, you compensate the answer by taking away the extra you have added." Another student, having caught up, will say, "Teacher, I

have another method, quite similar but still different. She makes 19, 20, but I prefer to make 27, 30. So I add 19 and 30 and later compensate by taking away 3." And other students will now jump in and bring out methods, which may be found in the textbook or not. And you see what we mean by creativity? Ability to see different ways to do one thing. But the mathematics is a basic skill of action. In our textbook, we do not have a dichotomy between the two because we need the creative method to really focus on the basic skills. We do not see creative methods as an extension of the basic skills. This could be one reason why our students are very strong with the basic skills. The creative activity is not creative as seen outside the curriculum. So this is what I mean when I say I do not see the dichotomy between the two. The concept of creativity, I just mentioned to you, is not our own invention. It's everything gray in the East Asian culture. So although I will describe to you what you will see in the Singapore textbook, you will see the same thing in a Korean textbook, in a Japanese textbook, in a Chinese textbook, in Taiwanese textbook, and in Hong Kong textbook, all these are countries where the East Asian or Confucian culture is very strong. And somehow, they become reflected in the way we teach. There is this concept of teaching creativity. Step 1, learn the basic learning vigorously. That is one way to do the basic and everyone must learn that. Enhance the rigidity that is often associated with the way we teach. We want every child to master that one way in each level. Next level, you modify it another way. Third level, you will depart from that one way. So this idea of learning one thing, modify one thing and depart from one thing. It is in all the literature, in different languages, focus on Confucianism. This is very different from the Western idea of creativity.

Guest: I have a question regarding critical thinking, creativity, computational skills, and balance of teaching and learning mathematics. What I mean by creative and critical thinking is the ability of the children to be able to integrate the mathematical concepts with the real world. Do you have that in your Math curriculum in Singapore? For example, conceptualizing problems on your own, in a collaborative activity, and be able to compute it in such a way that it will be very interesting to the children.

Dr. Yeap: Our textbooks have this section on problem posing. So you will find sections in the textbook that ask the student after solving some problem to make up problems. They get a problem in a group, and ask their friends to solve them. So there are some of these elements as well in the textbook. The exercise on critical thinking, earlier in the day, when I was solving

problems, do you notice I was demonstrating a critical mindset? As I read, I ask myself, "Do I understand?" "Is this easy for me?" "Oh it's difficult, what should I do?" So that's what we mean by more of the critical thinking and creative thinking, and generally the human quality aspect of it, this time, in the day-to-day teaching of the basic skills and word problem solving. But you will find that it will even describe by asking children on how to make up problems and connecting to the live kind of thing in the textbook. Maybe I should say one more thing about connecting Mathematics to real life. All over the world, most of us know what that means. In Singapore, we have a slightly different interpretation of real life. Most of us, when we say, use mathematics in real life, we taught about using mathematics when you go to the supermarket, use mathematics to do something that you are familiar with. In Singapore, we also interpret thinking, as what you do in real life. From connecting mathematics to real life means you think all the time. And because you think everyday, in Mathematics, you also do that. So that's the second interpretation of connecting Mathematics in real life: that we do not do things blindly, we always think.

Guest: For example, in an hour of Math class, what percent is going to be used for critical thinking and what percent will be used in the discussion of the concepts?

Dr. Yeap: Technically, I would say 100% is used for developing the human qualities. Because even in the teaching of basic skills like I mentioned, the teacher will always be asking them questions. So we are asking a more clear-cut question of how much time the teacher will teach the basics and how much time to use to do the problem solving. Then my answer will be, in the first four grades, the proportion is probably 7:2. In the fifth and sixth grade, the proportion is probably more on problem solving, more like 2:3.

Dr. Decenteceo: How often do you change your curriculum? Are changes driven by research on the outcomes of implementation?

Dr. Yeap: So just a recap of the curriculum. We introduced this revised curriculum, the problem solving curriculum, in 1992, fifteen years ago. We have not changed since. Today, we are using the same curriculum, the problem solving curriculum. The word education reform is hardly ever used in Singapore, maybe because we don't have it. We don't have drastic changes in our system. We are a very unexciting system. We do not have reforms. We have something that we came up with, which is small and

ambitious than before. That means, prior to the introduction of the problem solving curriculum, we have a very traditional way of looking at Mathematics, procedural and memorizing like everyone else in the world. We did that because we didn't know better. Then we introduced the problem solving curriculum in 1992, we don't make a big show of it. We put it in the system and we don't stay at it. So some teachers will look at it and say, forget about it. They ignore it. Some teachers say, "They don't understand, but let's find out." And they work on it. And by design, they are covered by the Ministry of Education and the university, and they attend. In other words, then what you call educational reform happens, even if you don't respond. You can do it. There's a very small sheet. For the next five years, nothing much happened. Except more and more people became used to the new idea of problem solving. And then, in 2001, almost 10 years, right? There is a revision in that curriculum. Why did we revise the curriculum? After 9 years, 8 years of implementation, we researched what was going on, whether published or not. We looked at how the students were performing in the national test and we thought, why have we done well and what have not gone on well as far as the total reform is concerned? For example, we introduced problem solving in 1992. By 2001, we found out the heuristics that our students are exposed to problem solving. But they are only used to strategies; they used the drawing that I showed you just now, the so-called model method. Why? Because the textbooks have them. And they used calculation. Why? No surprises, because the textbooks have them. In fact, they don't use any other method. So in the year 2001 when we revised the curriculum, the emphasis of that revision is, can you teach your students more methods? So more heuristics were introduced in 2001 and it became explicit. The new textbooks were rewritten to include more heuristics of methods than the tools that appeared before. So you get the idea. We get what is happening in the national test and do research to know what is the missing link. And we now proceed with the next revision. We cannot do anything explicit, or we do not do anything. So we do one thing. In 2001 to 2007, again we looked at research studies, we looked at how the students performed in the national test and we realized one thing. Oh good, now they use many, many methods in their answers. But it seems that the human quality act is still lacking. In particular, our students are not so good in geometrical thinking, problems that require visualization. We noticed that they have not performed well in the national test. We looked further at the TIMSS study. Although our students did well in geometry, they were not doing as well as they did in other tests. They are good, but compared to other strategies, they are not doing well. So these are all based in the 2007 revision. Guess what, the

Open Forum Highlights

81

teachers were told that in the revised edition, one has to focus on geometrical figures. There are other things which we have not done yet. But now we focus on one more thing, that is a formal curriculum review every six years. We have informal one in between, so technically, every three years we look at what is happening. And every six years, we will tell the teachers, "Okay, the next time around we've done all these very well. Our students are good, but can we do something else?" The message I am trying now to deliver is that we don't have reforms. We do a little bit at a time because that's more manageable.

Dr. Gureng: These two questions can be put together. One is about students who are really good in Math and those who are slow learners. When you give problem solving, students who are categorized as slow learners, there's also some problem about comprehension. So do you have any remediation for slow learners or any other papers about differentiated teaching? Probably it has something to do with the grouping of students.

Dr. Yeap: I'll start with differentiated instruction. Differentiated instruction is quite new to our teachers as well. And our teachers often ask, "How do I cater to my mixed-ability class?" There are 40 of them, and of course, they are of all sorts. How do I cater to them? The Western literature often say that you teach them differently. You don't expect so much from the weaker students. And you accelerate the better students or you enrich the better students. It's something we don't subscribe to in Singapore. Because we need to develop every single child for economic reasons. If 1/10 of our children perform well, we survive. Now, if 1/10 of our children perform well, ten years from now, we wouldn't survive. As a region, we need 3/4 of them to do well. It's urgent. But we seem to be able to cope with that. About 90% of our students are doing all right. Differentiated instruction to a Singapore teacher is, "How do I teach everybody so that they all become fine achievers?" That's the concept of differentiated instruction in the teacher's mind. In fact, for the weaker students, we let them do the easier things. The better students, challenge them. We do not subscribe to that and we do not criticize. Every student can do the difficult things. But we use different methods. So in differentiated instruction, we agree on a specific example because this example tests their skill. In the second grade class, the teachers are asking the students to do a page of exercises on one section. And the teacher asks the class, that class is about 40, "Boys and girls, let's do the first half together." While the students are doing the first half, this teacher, walks around and looks at what the students are doing. Two

minutes, she comes to the front and says, "Very good. All of you seem to remember what we learned vesterday. The four of you are still struggling, that's all right, I'll come to you in a while. But I noticed that John used a different method than most of you." And the teacher will say, "John can you tell the rest what you did?" And John struggled to put what he did into words. John is a high achiever. But he seems to be struggling in articulating his thoughts. This is differentiated instruction. But John is already good in computation. You cater to that by making them do something differently. And John, after some difficulty says that, "Essentially, you are taking away 200. And then do some compensation." And then the teacher will say, "Sharon, do you understand what John is saying? Can you try to explain it more clearly than how John is saying it?" And Sharon tries, Sharon struggles. The teacher thought that Sharon is also okay in doing the computation but she thinks she is not so good in articulating her thoughts and the teacher is now making her do what she is not good in. And the teacher says, "Okay, look at the other eleven questions. Can you draw a circle around those that you think you can do John's method? Go on." While the rest of the class think of other calculations that they think they can do, the teacher now remembers there were four students who were struggling with the basic computation. And very quickly, she calls the four of them to the front. And in two minutes, she tried to help them with success and sent them back. And by then she says, "Oh, which one of you think you could have used John's method?" And before she said the possible solution, she told them, "In the next part, while we do the computation, the rest of you try to finish the ones that you didn't finish yet. And when you're done, you go back and join the class." Can you see how the teacher is catering to students of different abilities? Those who are weaker, call them to the front, give them on the spot remediation. Those who are good but they are not good in articulation, she gave them the practice. And the rest who can do the calculation, she asked them to think of using a more creative method. That's differentiated instruction. We also have remediations, which are more common. For example, after school, at one o' clock, the teacher may ask six of them to stay back and conduct an informal session, maybe gather outside the class to teach them one specific or two specific things that they could not comprehend.

Dr. Gureng: There's a question here about checking papers, deciding points, and there are two situations presented. Supposing there are two problems here. Somebody's computation is indicated in the usual process and another student provided an answer to this without anything written before that. How do you evaluate this situation?

Dr. Yeap: These are two separate things. That is what I call mapping objective during examinations of formal assessment. We do not give points during their daily work. It is not necessary. We will give you points when we are grading test, or formal examination. We have two long examinations given at the end of the year. And we might have an intermediate test in between. To put it simply, we have four formal assessments in a year, where we give points to the students. What if the student didn't show the method? Two answers to that. If this is problem solving, and you know in problem solving, it is probably complex and it is probably very new. So if they are able to solve the problem, very good. That is, you see things that other students cannot see. Therefore, you are responsible to execute the method you employed. If you are able to solve it but you are not really or not able to execute your method, you are not good even though you can solve it, even if you can see the answer and you write down the final answer. If you are not able to explain how you did it, then you get no credit for it. Even if you are brilliant, and fully understand the lesson, they won't think you are brilliant. And we will not give you credit for it. So in problem solving items, if the student shows the method, and the final answer is wrong, they may get a four out of five. They get penalized for incorrect answer, but four points for the method. And if they get a final answer without working, they will get zero point. This is to distinguish between brilliant ones who can communicate but with some small final computational error. But those who are not good in explaining the method in a new situation would not be credited. So in problem solving items, there's a lot of emphasis on communication. Whether you're brilliant or not is another point altogether. You can be brilliant, plus able and willing to communicate. But if it is a basic problem and basic application, like "sixty five cents, four hundred grand, two kilograms, how much?"-these are so mundane, and so common-if others couldn't understand you, it's their problem, not yours. So in those situations, you are allowed to just give a final answer without explaining anything. To put it very simply, if the problem is basic, then you can just give the final answer. But if the problem is new or complex, you are expected and have the responsibility to explain yourselves. So that's Math examination. Obviously, the learner will answer to a very lengthy process, to explain yourselves.

The second question you asked is "how much time does the teacher spend checking the student's work?" Too much time. If you are a teacher in Singapore, like teachers everywhere, you will probably complain and say, "After school I have to check 40 students' work, and if I have 4 classes, I have 160 books to check." They do this everyday. And sometimes to the extent of their own personal time. They do not give points. They just see what the students do so that they know what the students remembered.

Dr. Gureng: This is about politicians' involvement in the curricular operations of Singapore. I will read it the way it's written. The Singaporean educational system, as I understood, is a system independently run by people. Does this mean that the politicians have 'no hand' or are 'hands-off' in the structuring of the system?

Dr. Yeap: The education system is largely run by the government. We only have a public school system. So technically, everything is under the government. The government is made up of a bunch of politicians. So technically, our education is run and supported by the government. But fortunately, our politicians know when to stick their fingers in and when they should not. We have our Minister of Education, he is a politician. He was voted to serve the government. But you also know that in Singapore, our government is very stable. We agree, the Minister of Education is a politician. But everyone else in the Ministry of Education is not a politician, they are educators. So he is responsible, he makes the final decision but is smart enough to know that he's not smart enough to know everything. So you have different people to do different things. So ours is a structure that is largely composed of educators but is headed by a politician. But our politicians are different from politicians everywhere. Again, this is probably a different discussion of politicians. Our politicians are very mild people. They don't yell at each other, they talk nicely. Because people have different opinions, our opinions are very flexible.

Dr. Gureng: May we know the number of teaching load of your teachers in Singapore? So if you are a teacher in Math in that school, how many classes, how long, how many preparations, are you going to teach in Grade 3 math, Grade 6 or Grade 5?

Dr. Yeap: In elementary school, teachers are not specialists. That means they are expected to teach almost everything. So you're probably the Math teacher, the English teacher, you're probably the health education teacher. Everything. What you won't be is a Physical education teacher, that's a specialist. Nor an Arts teacher, that's a specialist. Nor a Music teacher, that's a specialist. You're not the whole language teacher. Other than that, any subject could be what you'll be asked to teach. So I teach Mathematics, I teach English, I teach Science in elementary school. In secondary school, we are trained to teach two subjects. So finally, you'll be

Open Forum Highlights

teaching two subjects, could be Mathematics and Physics, could be English and Mathematics. Two subjects. So that is what you do as a teacher. It's not per hour. Our schools have very short hours, you probably know about that. Our elementary school opens at 7:30 in the morning, up to one o'clock. After 1:00, the students have to get out of class. Definitely they go home. Sometimes, they have activities-Math club, could be sports, basketball, could be some special activity. But not everyday, some days, they go home straight. That's because the first and second graders will come to the class at one o'clock in the afternoon and will study until 6:30. We are a system where two groups of students share one room. We had to do that a long time ago when we first started because we were very poor, and we did not have enough classrooms. So what is the solution? So that started the afternoon session. So, 7:30 a.m. to 1:00 p.m., 1:00 p.m. to 6:30 p.m. This style is no longer the operation for secondary school. The secondary school comes in the morning and will end at 1:00 p.m., 1:30 p.m., 2:00 p.m. and the latest, 2:30 p.m. What happens after the official time varies. But these are not formal activities anymore. It could be non-academic activities like games and sports, and other activities. It could also be a remediation that the teacher arranged to be enjoyed by groups of students. Now they leave at 1:30 p.m. to 2:00 p.m. You, as a teacher, will be running around like a headless chicken. That is, you hardly have the time to sit down and have a cup of coffee. You will teach this subject, you will teach that subject and, in the next period, you have to run to another class. You'll be running around. Then there's the recess. If you are a teacher with youngsters in the elementary school, some days you must be in the pantry with the students to supervise the rest. Other times, you can finally sit down to put up your legs. To put it very simply, during the official time of teaching, teachers of all levels are running around, busy. Sometimes you even forget to say good morning to your colleagues. After the official hours, there's no time to relax. You can spend time with your students. You might have meetings that you might want to attend and a variety of activities. Some days you are free. So that's the typical life of a teacher in Singapore. I don't think it's different anywhere.

Dr. Gureng: I'm going to read the next questions one after the other because somebody mentioned that you have been requested to show us the video of a classroom in Singapore. While preparing for that, may I read the following questions. "What is the extent or involvement of your high school teachers in the preparation of the scope and sequence?" The other one is,

"how is Math taught in the high school? Is it spiral like a few of everything? Is it compartmentalized, wherein they have algebra, geometry, and so on?"

Dr. Yeap: In high school, we have an integrated program. We are not Americans, so we do not follow the Americans. We teach the British way. Mathematics, you know everything about Mathematics. So in high school, that's more Algebra subject or Geometry subject. It's all in one book. And it's integrated. So that's for the curriculum.

Dr. Gureng: And about cooperative learning, and strategies, approaches. The other one is, "when is the mastery of a particular subject matter attained? Is it 75% passing? Or 95% passing? How many students should get into that, particularly if there's a certain level of mastery that you know already?"

Dr. Yeap: By mastery, we mean if you get 25% of the score, you have mastered the basic skills. So if all you can do is mastery, you get 25% max, obviously you have mastery, you learn 25%. If you can do basic application plus basic skills, you'll get about 50-60 percent. So minimum mastery is basic skills and basic application. The last 40% is problem solving. 20% complexity and 20% novelty. So if you get about 20-30 percent of the score, in the mind of the teacher, this means you're ok. Anything else is not. You get about 50-60 percent max and you mastered the basic application as well. And that is certainly quite good to proceed to the next level. Strong enough foundation to follow up. But if you can go beyond 60 or 80, it means that your problem solving skills are ok. It's happening. And if you can go beyond 80, it is good. You are very good in the system. That is as far as what you can expect from mastery. But we expect, as unreasonable as it sounds, we expect up to problem solving. What does the system expect? The system expects mastery of the basic application. So if you can do basic skills application, the system says you can go forward. But the teachers will tell you, "Oh, that's not good enough. John, you're getting 60 in Math, and you should try harder." And that is, the teacher wants John to do problem solving. So the system expects basic application and mastery. But the teacher also expects everyone to do problem solving. And to try different things. Singapore schools are encouraged to find their niche area. So if you speak to a Singapore teacher, you ask the teacher, "Your school, what is their niche area?" They'll tell you, "Oh, my school's niche area is statistics." That means they have extra funding and they focus on their niche area. Some will say, "Oh, my school's niche area is academic excellence." That means they focus their

Open Forum Highlights

attention on producing things. And another school will say, "Our niche area is aboriginal acts." We send our teachers all over Australia, Europe, everywhere, to learn about aboriginal acts. And they come back and they train other teachers. And the teachers and the students are exposed to the aboriginal acts. So different schools have different niche areas. Some schools put their niche area for cooperative learning or multiple intelligences and so on. So all these things are happening but not in the same degree as to other schools. It depends on what the principal is passionate about, what the leaders in the school are passionate about, etc. Thank you very much. I will end the session now by hoping that whatever is not clear by my words will be made clearer by the video. What I'll show you is my first grade class. Last year, I went to a school to teach for one year, as I do, from time to time. Although I train teachers mostly. And this is what you will see in a typical first grade class. I like to teach the first grade because they have very little language and they're still new. And we see what they do. The lesson is on the addition of single digit numbers. So, the Singapore textbooks have one section where they learn how to add three single digit numbers, and a day before this lesson, they have just learned that. And in this lesson, they are trying to solve a problem. So the lesson is to focus on consolidating the ability to add three single digit numbers, at the same time, getting them to solve a problem. So I leave you in the good hands ...

(after the presentation)

Before we finish, is there any question you would like to ask that is not clear in what I've shown you? So that is the kind of lesson we want our teachers to teach in class since many of them can't teach this kind of lesson easily. This is one of the lessons I conducted with my students last year. Maybe you want to ask anything that is not clear. You want to know about technique? Anything about the Mathematics, or anything about the classroom evaluation. And you probably realize I was doing assessment all the time. Although it was not testing, I was doing a lot of assessment during the 20-25 minute segment of the lesson. Our one lesson is 30 minutes. And one hour again. So after this part, we go back to individual test. And they do some written work based on the activity they just did. But any question that you would like to ask, or any information you want to ask about? Anything else? You need a lot of patience to go through lessons in that manner. So I always respond to that. In a class, where we train teachers, sure they must know their Mathematics to that extent. What is important to teacher training is we give them the right paradigm to their role. Why are they there? Are they there to catch a train? Are they

Open Forum Highlights

rushing for something? What's the hurry? In answer to that, I demonstrate to my trainee teachers that you don't have to rush. There is no rush. Or maybe there is a rush. But if you rush, who are to be left behind? And you must come back and get them. Would that mean more time? Do you want to do that? So in my day, in my time, when I'm teaching, I don't rush it. Sure, I know the curriculum to cover. But if you don't rush, you don't leave them behind, later they'll go ahead of you. At some point, these kids will be much brighter than us. As long as you don't leave them behind. Once you leave them behind, they cannot do many things. So I take a lot of time. Actually I was teaching them a lot of things. Mathematics is any two digits or three digits. But the basic of it is we are teaching them how to think for themselves. We are teaching them to remove their biases. You are teaching them how to speak up. We are teaching them so many things. And you do these a few times. And anything else, you teach them very quickly. So you take a lot of time, a lot of patience. But what's the rush? Some of you may want to respond to that? Anything else you need to ask or want to clarify?

Guest: Good day. I am just curious. In our setting, we usually prepare lesson plans. And in our lesson plan, we have the format wherein we have the preparatory activities, the motivation, the discussion proper, and then the application, so on. But it seems that in your case, it's different. Is that the only flow of the lesson you are presenting on a particular day?

Dr. Yeap: In Singapore, we also teach our students lesson planning. In this lesson, you see the preparation. The first few minutes, I was talking to the children. "What are you doing during the holidays?" The first part of the lesson, what we call the 'introduction' or the 'preparation.' I'm warming them up to speak. They don't speak naturally because they are not native speakers of the English language. I will ask them something they can answer and speak about naturally. So I ask them, "What are you going to do during the holidays?" I was warming the class up so that later they will be able to articulate their thoughts. So that in the traditional method plan, it is the introduction of the lesson. After the introduction, the second segment is preparing for the lesson. In this segment, we are preparing for the showing of the magic. I will show them the magic, I won't explain. I will just show them the magic. So that is the first part of the activity. The main development is they can add and later they can see the patterns. So two things; they can add, they can see the patterns. That is the major part of the lesson. At what part of the lesson do you realize to add those two numbers? How much? Sometimes to the whole class. Sometimes to a specific pupil. And that is the male one discussing to see the pattern. That took a long time

89

because they could not see the method. That would be the development part. The last part is when they consolidate what they have learned or they explain what they have learned. That one is after the 25 minutes when they go back to their respective desks. And they are given a worksheet where they do more of this and practice adding the numbers. So although it does not seem to fit in the traditional format, actually it is designed in the format you have described: introduction, setting up the activity, development, consolidation.

Guest: Thank you, sir. I think it is more relaxed to teach in Singapore.

Dr. Yeap: It's up to us really. Any of us can actually teach this kind of lesson anywhere. And I can tell you, any children will respond in this way. I have taught this lesson in Chile, where they only speak Spanish and I only speak English. And we have this lesson. And the Chilean teachers tell me, "Oh, our students cannot do this." And so I said, "Let us start doing and see whether they can or cannot." Of course they can.

Dr. Decenteceo: The curriculum imposes some constraints on instructional time. In this regard, how much variation is acceptable? How much leeway is given to teachers in terms of topic coverage?

Dr. Yeap: The limit, we all have to teach the entire curriculum, that is, to finish the entire book. By the end of the year, if I don't finish the book, I am in trouble. So I must finish the curriculum. That goes without saying. And every teacher will finish the curriculum, and they can. What is a leeway? You can do anything you want in a classroom. So long as the pupils learn. All schools will have a general direction. But once you are in a class, you don't want the students to wait. You are in control. In this case, I have chosen this activity, not I, I should say 'we' have chosen this activity as a lesson to consolidate the adding of three one-digit numbers. According to the scope and sequence, according to the scope, they must learn how to add three one-digit numbers. According to the sequence, they have learned this, and today is one more lesson to consolidate what they have learned. I have decided to bring out a worksheet and make up more mini-exercises and give to them. Or maybe not. Or I can tell them about the lesson or I can do this. I could have chosen any other activities to consolidate the addition of three single-digit numbers. I could have done one way or the other. So that's fine with me. And last year, I finished that curriculum. At least managed to finish the curriculum. I will not rush it. It's part of the scope and sequence. In Singapore, we do not think of substituting hard work where we do what we do and, in this way, what we are supposed to do is adding three one digit number. That's one lesson for learning, that's one lesson for consolidation. The day before that, they were learning, they were just adding up numbers, three single-digit numbers. And today, we are consolidating. And you noticed one boy at the end who couldn't add three one-digit numbers? What is his problem? I will assess it. I will start by saying, because one student couldn't add, "Why couldn't you add?" When I give him 2 numbers almost immediately. But when I say, "Give me the next one." He couldn't do it. So 2 plus 4, you get 6. Six plus 8, he couldn't do it. What do I do? I really got him on the spot a little bit. What did I do, you noticed I was showing my fingers. Clearly, I only have five fingers. So he couldn't count all my fingers to get the answer. I was actually showing him 8 on my fingers. So I was telling him, "You know, Aisel, you got 4 and 6 already, count 1 to 8." So he probably got the idea. Oh, 6 then he looked at his teacher's finger, 7, 8, 9, 10. And you count the number of times he got the answer and corresponded to counting my fingers, but without looking at my fingers. You're counting just the same. So that is a simple remediation for me. I lead you in class, why did I lead you in class? Because of 2 things. I want to give them a break. It's a testing ability. They are already tired. So I want them to just be relaxed. The second reason is I want them also to be aware that although most of them could do so easily, there are some who struggle. And we need to be able to realize that not everyone think at the same level. Someday, sometime it could be you. You're not thinking at the same level as your friend. That is why it's very important to make yourself very clear. Even on the day you get it, there will be others who won't get it. If you get it, so does John. So John will make it clear to other people. So I direct them all to their seats for themselves. And someday it's worse, someday it's you. And you need to empathize in any way. Anything else you want to ask? So I think I better stop here because it's time.

Afternoon Session

Ms. J. Evasco: May we call on Dr. Gureng to be the moderator again? For those who have questions, we will follow the same procedure this morning. May we ask Dr. Yeap to join the panel onstage. Thank you.

Dr. Gureng: This is the first question: How can we, it refers to us teachers, eliminate or minimize procedural errors among our students, in doing the Mathematics? I feel if we can get rid of these errors or types of errors, we can be at par with Singapore.

Dr. Yeap: It is not surprising that students make procedural errors. Because our human mind is never designed to carry out procedures in the first place. If you want students to learn procedural knowledge, you will expect them to make procedural errors. As teachers, let's teach them extensively in this area. If we just teach students procedural understanding, they are just clearly inadequate. We say that we must teach them what we call relational understanding. I think the second speaker makes some reference to that as well. So all procedures must be taught with the corresponding conceptual understanding. If procedures are taught in the absence of conceptual understanding, you will expect them to make the errors.

Dr. Gureng: Well answered. That's right. We are emphasizing so much on algorithm. And in that process, we can see deeper forms for one student to the other. And when we see these things, we put a line after seeing something erroneous after that.

Guest: Sir, I am David Esteban from the Esteban School, and I'm in the school administration. It just occurred to me that we're dealing with two different countries, of course, that will have two different systems, two of measuring the performance of the students and we're also dealing with the TIMSS which is very standard. Am I correct in understanding that? Is there an effort in aligning the two? First, is the Singaporean assessment made with the TIMSS in mind? Because there's a thing as teaching to the test. And the second thing is, does the national assessment for the CEM, design their tests with TIMSS in mind?

Dr. Yeap: TIMSS is not well-known in Singapore. I can tell you that more than half of our teachers have no idea what TIMSS is all about. TIMSS is really not popular in Singapore. When the results are out, they just wait for

Open Forum Highlights

the report. There's no discussion about it. Nobody discusses it in Singapore. It's just a benchmark for us to know how we do in comparison with the other countries. What are our concerns in Singapore? Our concern is, how come this group of student is not learning? How can we make our average students perform better? How do we challenge our best students? The 10% are struggling, what can we do? How can we make them as good as the rest? Clearly, our national test is never designed with TIMSS in mind. Neither is our classroom teaching. Most of us don't know what TIMSS is.

Dr. Gureng: So Singapore worries about the remaining 10%, the Philippines worries about the 90%. I think the other question is, does the CEM test also have TIMSS in mind?

Guest: The Singapore benchmark internationally. I was wondering, is there a movement as well for the Philippines to benchmark internationally? That's really the core of the question.

Dr. Decenteceo: CEM's testing program is not based on TIMSS or any other large-scale test focusing on national or international comparisons of student achievement. I am also not aware of any benchmarking efforts being done locally. But I must say that CEM has taken a step to look into benchmarking. Through a partnership with the Australian Council for Educational Research, CEM administers exclusively ACER's relatively new program – the International Benchmark Tests (or IBT) in English, Math and Science. The tests are not curriculum-bound. The Grades 4 and 8 tests in Math and Science are TIMSS-related. IBT benchmarking is done with countries who participate in this program. As we expand the Philippine database on the IBT, we hope to be able to establish more solid benchmarks in English, Math and Science at different grade levels.

Dr. Gureng: The one mentioned by Dr. Yeap is about the public school system, the basic education public school system. And then he is talking to us, private school teachers, who are trying our best to raise that. But incidentally, I don't know what percent of students from private schools are going to be randomly taken to consider the group of TIMSS test. If that will happen, that will just be too minimal because the TIMSS group is going all over the country. They stop only in some places and they have a certain number or percentage of students participating in those places. We cannot even volunteer our students to undergo the TIMSS.

Dr. Decenteceo: I think we need to know more about Philippine participation in the TIMSS. TIMSS focuses on public education but I understand the private sector is involved to some extent. In any case, generalizations from the TIMSS results about the state of education in the Philippines can be misleading. Perhaps we can start a debate on the topic.

Dr. Gureng: Dr. Yeap, do you have any example of a TIMSS item? Like, Dr. Decenteceo says that it is generic. So that is using trapezium or trapezoid? If it's using trapezium, no Filipino can get the answer.

Dr. Yeap: The items are actually translated into the local languages. So for example in Thailand, they do it in Thai. In Singapore, they do it in English. The terms also apply to the Philippine setting, it's going to be the American terms. In Singapore, it's going to be the British terms. So the items are not identical for everybody. The basic items are the same, but will be translated in the local terms or local language.

Dr. Gureng: If we are teaching so many topics per grade level, then there's too much to cover per grade level. After that we expect much from it among our students. Does CEM have the influence in DepED to make changes in its expected learning comprehension for each grade level?

Guest: Learning competencies, I was the one who asked. Because it is what we know or we just assumed. I was the one who asked the question because our Mathematics teachers are having difficulty trying to cover all the topics within that one school year. And we find that the children really want a more in-depth understanding of a concept especially in grade 1 or 2, and then the teachers are, you know, *humahangos sila kasi* they have this certain number of weeks to teach the topics so they have to move on. And it's very difficult to provide for remediation for the children who are left behind. So I was just wondering, does CEM have the influence in DepED so that DepED could make the necessary changes in the curriculum, the scope and sequence, the number of expected learning competencies per grade level? So there will be a quality change, since it is difficult to teach them, if they don't get the basics in grades 1 and 2, and you can just imagine upon reaching grade 6, the more they can't comprehend.

Dr. Decenteceo: Our test development efforts begin with the Basic Education Curriculum developed by the DepED. That curriculum helps us define the coverage of the content standards and skills measured by our tests. When DepED revises the curriculum, we adjust our tests. What is

referred to as "assessment-driven curriculum" could be the subject of joint efforts of the two institutions in order to attain a curriculum attuned and responsive to the needs of all stakeholders in education. This is clearly an area where DepED and CEM can mutually influence each other

Dr. Gureng: What do you do with misbehaving students? Do you give detention, suspension or some sort? Or there's none? No misbehaving?

Dr. Yeap: There's a very structured system within every school about pupil management. We have a head of department for pupil management. So pupil management is an important area of work. All our teachers are trained in pupil management and classroom management. Pupils who misbehave, they will face the consequences. And it all depends on the actual misbehavior, it's probably too much details. Generally, in Singapore, in the elementary school, you will hardly hear much discipline problem. What trouble can they give teachers? It's hardly a problem in elementary. But there are things put into place, more counseling rather than hitting the student. Nobody can hit the students anyway, except the head of the department of the pupil management and the principal. They can cane the students, they can cane the students in the offenses that warrant caning. In secondary school, the fact that they are teenagers, they also have problems. Some are personal. Others are family-related but hardly ever because of school. It's usually all the external problems coming in. But as a teacher in school, we have to view this as well and, similarly, there's a system or ways to help the students. Again, more counseling than punitive in nature. So probably, that's sufficient details.

LIST OF SCHOOL PARTICIPANTS

Abraham's Flock Christian Academy Sta. Rosa Uno, Noveleta, Cavite

Adamson University San Marcelino St., Ermita, Manila

Angelicum College 112 MJ Cuenco St., Quezon City

Antipolo Immaculate Conception School Sumulong St., San Roque, Antipolo City

Assumption Antipolo Sumulong Highway, Antipolo City

Ateneo De Manila University Katipunan Avenue, Quezon City

Blessed Mary Academy Dasmariñas, Cavite

Brightwoods School Angeles Citi Center, Pandan, Angeles City

Canossa College-San Pablo San Pablo City, Laguna

Capitol Hills Christian School Filinvest II Rd., Batasan Hills, Quezon City

Caritas Don Bosco School Laguna Technopark, Laguna Blvd., Biñan, Laguna

Children of Mary Immaculate College 22 Maysan Rd., Malinta, Valenzuela City

City of Mandaluyong Science High School E. Pantaleon St., Hulo, Mandaluyong City

Claret School of Quezon City Mahinhin St., U.P. Village, Diliman, Quezon City

Colegio De Sta. Rosa 6321 Estrella St., Makati City

Colegio San Agustin Southwoods Interchange, Biñan, Laguna

96

Colegio San Agustin Carissa St., Dasma Village, Makati City

Concordia College 1739 Pedro Gil St., Paco, Manila

Cradle of Joy Center For Learning Brgy. Sacred Heart, Kamuning, Quezon City

Creative Learning School, Inc. #11 Stella Maris St., Brgy. Kapitolyo, Pasig City

De La Salle – College of St. Benilde 2544 Taft Avenue, Manila

De La Salle Canlubang Brgy. Biñan, Biñan, Laguna

De La Salle Lipa Integrated School Mataas Na Lupa, Lipa City

De La Salle Zobel School Ayala Alabang Village, Muntinlupa City

Don Bosco Academy Mabiga, Mabalacat, Pampanga

Don Bosco School (Salesian Sisters) Inc. 3500 V. Mapa Ext., Sta. Mesa, Manila

Don Bosco Technical College 736 Gen., Kalentong St., Mandaluyong City

Don Bosco Technical Institute Sto. Cristo, Tarlac City

Don Bosco Technical Institute Makati Chino Roces Ave., Makati City

Elizabeth Seton School Bf Resort Village, Las Piñas City

Elizabeth Seton School - South Anabu II-D, Imus, Cavite

Esteban International School 2332 Pasong Tamo Ext., Taguig

E. Rodriguez Jr. High School N.S. Amoranto, Quezon City

List of School Participants

Falcon Learning Center Dahlia Avenue, West Fairview, Quezon City

Good Shepherd Christian School 71 I Lopez St., City of Mandaluyong

Grace Christian College Grace Village, Sto. Domingo St., Quezon City

Guadalupe Catholic School 1923 Orense St., Guadalupe Nuevo, Makati City

Holy Angel University Sto. Rosario St., Angeles City

Holy Family Parochial School East Capitol Drive, Kapitolyo, Pasig City

Holy Family School of Quezon City, Inc. 66 Maginhawa St., UP Village, Diliman, Quezon City

Holy Rosary Academy of Las Piñas City St. Joseph, Naga Rd., Pulang Lupa, Las Piñas City

Immaculate Conception Academy Greenhills 10 Grant St., Greenhills, San Juan City

Immaculate Heart of Mary College 54 Aurora Blvd., Quezon City

Immaculate Heart of Mary School Grotto Compound, San Jose Del Monte, Bulacan

Immaculate Mother School 872 St. Joseph St., Commonwealth Ave., Quezon City

Infant Jesus Academy – Antipolo Brgy. Dela Paz, Bagong Nayon II, Antipolo City

Infant Jesus Academy - Marikina Russet St., Rancho Estate IV, Marikina City

Ingenium Learning Resources Foundation, Inc. SSS Village, Concepcion Dos, Marikina City

International Christian Academy Fourth Estate Subdivision, Sucat, Parañaque City

Joseph and Mary Academy Rainbow Ave., Pacita II, San Pedro, Laguna

List of School Participants

98

Jubilee Christian Academy E. Rodriguez Sr. Ave., Cubao, Quezon City

La Consolacion College Biñan Sto. Tomas, Biñan, Laguna

La Salle College – Antipolo Town and Country Heights, Brgy. San Luis, Antipolo City

La Salle Greenhills Ortigas Ave., Mandaluyong City

Laguna Bel Air School Laguna Bel Air Subd., Don Jose, City of Sta. Rosa, Laguna

Lourdes School of Quezon City Don Manuel Cor D. Manuel Sts. Smh, Quezon City

Makati Hope Christian School Pasong Tamo Extension, Makati City

Malate Catholic School San Andres Cor. Madre Ignacia Sts., Malate, Manila

Malayan High School of Science Paz M. Guanzon Street, Pandacan, Manila

Manresa School BF Homes, Parañaque

Maria Montessori Children's School Foundation, Inc. Madrid St., Merville Park, Parañaque

Marian Learning Center and Science High School 41 D Silang St., Batangas City

Mary Help of Christians School, Inc. Mabiga, Mabalacat, Pampanga

Mary Help of Christians School, Inc. Acacia St., Cerist Canlubang, Laguna

Mary the Queen Academy of Pampanga Cabalantian, Bacolor, Pampanga

MGC New Life Christian Academy University Park, Global City

List of School Participants

Miriam College High School Katipunan Rd., Loyola Heights, Quezon City

Montessori De Manila BF Executive Village, Las Piñas City

Morning Dew Montessori Bo. Halang Marcos Hi-Way, Cainta, Rizal

New Horizon Integrated School 1Ferrer Compound, Brgy. Pinagbuhatan, Pasig City

New Horizon Integrated School - Makati #30 Lapu-Lapu St., Paseo De Magallanes, Makati City

Notre Dame of Greater Manila Madre Ignacia Ave., Grace Park, Caloocan City

Olivarez College Dr. A. Santos Ave., Sucat Road, Parañaque City

Our Lady of Caysasay Academy Taal, Batangas

Our Lady of Fatima Parochial School Mag-Abad Santos St., Bacood, Sta. Mesa, Manila

Our Lady of Guadalupe Minor Seminary Edsa Cor. Bernardo St., Guadalupe Viejo, Makati City

Our Lady of Pilar Montessori Center Silver Cor Bronze Sts., Pilar Village, Las Piñas City

Our Lady of the Holy Rosary School Deborahville Subd., Amaya I, Tanza, Cavite

Paco Catholic School Paco, Manila

Pasig Catholic College Malinao, Pasig City

Philippine Pasay Chung Hua Academy Luna Street, Pasay City

Philippine Cultural College 175 8th Ave. Ext., Grace Park, Caloocan City Philippine Cultural College 1253 Jose Abad Santos St., Tondo, Manila

St. Bridget College Batangas M.H. Del Pilar Street, Batangas City

St. Vincent De Paul College Meadowood Executive Village, Bacoor, Cavite

San Beda College Alabang Alabang Hills Village, Alabang, Muntinlupa City

San Beda College Rizal Highlands Pointe, San Juan, Taytay, Rizal

San Benildo Integrated School Marcos Highway, Cainta, Rizal

San Benildo Integrated School Sumulong Highway, Antipolo

San Jose Academy M. Naval St., Brgy. San Jose, Navotas City

School of Our Lady of La Salette Muzon, San Jose Del Monte, Bulacan

Second Mom-Domuschola Internationalis Molave Street, Ugong, Pasig

Siena College Del Monte Ave., Quezon City

Southville Woodland School Liwayway Street, Sta. Ana, Taguig City

St. Andrew's School 475 Quirino Ave., La Huerta, Parañaque City

St. Bridget's School 1047 Aurora Blvd., Quezon City

St. Dominic Academy Pob. Pulilan, Bulacan

St. James Academy Sta. Ines, Plaridel, Bulacan

List of School Participants

St. Jude Catholic School 327 Ycaza St., San Miguel, Manila

St. Mary's Angel Academy Abangan Sur, Marilao, Bulacan

St. Peter the Apostle School 1260 President Quirino Ave., Paco, Manila

St. Stephen's High School 1267 G. Masangkay St., Sta. Cruz, Manila

St. Therese Martin of Lisieux School Sta. Cruz, Laguna

Sta. Catalina College San Antonio, Biñan, Laguna

Statefields School, Inc. National Road, Molino 3, Bacoor, Cavite

The Learning Tree Child Growth Center 134 V. Luna Ext., Sikatuna Village, Quezon City

University of Sto. Thomas High School España, Manila

Veritas Parochial School Ph 1, Bf Homes, Parañaque City

World Citi College 461 William Shaw St., Caloocan City

Xavier School 64 Xavier St., Greenhills West, San Juan City

Emerging Issues Mathematics Curriculum

What we know . What we need to know



Educational Measurement, Inc. Symposia Series

A CEM Symposium

3rd Floor, SGV Hall AIM Conference Center Manila July 17, 2008 Program Thursday, July 17, 2008 8:30cm – 4:00pm

MORNING (8:30am - 12:00nn)

Opening Prayer

Sr. Monica U. Navarro School Principal – Immaculate Heart of Mary School

Welcome Address

Dr. Lenore Ll. Decenteceo President - CEM

Keynote Speech:

An analysis of mathematics items in national examinations in Singapore Dr. Yeap Ban Har Assistant Professor - National Institute of Education, Singapore

OPEN FORUM

LUNCH (12:00nn - 1:00pm)

AFTERNOON (1:00pm - 4:00pm)

A comparative analysis of the elementary mathematics curricula in the Philippines and Singapore

Mr. Jason V. Moseros Test Development Assistant IV - CEM

Error pattern analysis in mathematics: A springboard for intervention Ms. Ma, Angeles A, Sampang Test Development Officer II - CEM

OPEN FORUM

Closing Remarks

Ms. Esperanza C. Buen Vice President for Operations - CEM

000

Moderator

Dr. Paulino Gureng Special Assistant to the President - La Salle Green Hills, Mandaluyong

Reactors

Dr. Rosemarievic Villena-Diaz Professor – Philippine Normal University

Dr. Lynda P. Parreño

(registration starts at 7:30am)

PUBLICATION GUIDELINES

The Center for Educational Measurement, Inc, (CEM) has always recognized the efforts undertaken, individually or collectively, by the teachers, principals or guidance counselors, particularly in the areas of educational measurement, assessment, and evaluation.

It is the intent of CEM to support these efforts by inviting schools to submit articles or research papers to our official publications, the *Philippine Journal of Educational Measurement* (PJEM) and *The CEM Standard*. However, to facilitate publication of papers in printed form, we request the authors to adhere to the guidelines detailed below.

A. The Philippine Journal of Educational Measurement

Description

The *Philippine Journal of Educational Measurement* (PJEM), a refereed journal, is published annually by the Center for Educational Measurement, Inc. The journal aims to contribute to a better understanding of the system of measurement in the field of education across all levels basic to higher education in the Philippines. As such, the journal contains empirical and nonempirical reports such as theoretical studies, research studies, evaluation studies, specialized reviews, essays, reflective inquiry, critical book reviews, commentaries related to educational testing, measurement, assessment, evaluation, and research on substantive, innovative, and methodological issues.

Article Content

The PJEM welcomes contributions from teachers, researchers, measurement theorists, school administrators, policy-makers, and other key stakeholders across all levels. All articles are accepted on the basis that they are original materials, have not been previously published, and are not currently under consideration for publication elsewhere. Articles may fall into the following categories:

 Theoretical studies. These are studies concerned with the development of theory in the analysis of measurement in order to enhance the understanding of measurement processes.

- Evaluation Studies. These are studies that assess the extent of implementation and impact of a specific program or project and usually emphasize needs assessment and/or ongoing feedback to program implementers.
- Research Studies. These are studies that typically use data derived from qualitative or quantitative methods or both, including but not limited to, experiments, case studies, surveys, philosophical investigations, and historical studies, to yield new information, to focus on specific projects or settings, or to synthesize emerging patterns.
- 4. Specialized reviews. These are articles aimed at critically relating issues, comparisons, and analyses to the application of educational measurement and models in the educational process, but well founded in the existing literature. Reviews focused on research, theory, methodology, theme, theoretical contributions, critiques, and instructional techniques are accepted.
- Book reviews. These are articles critically evaluating the strengths and weaknesses of a book relevant to the scope of PJEM. Books reviewed must be within two (2) years of its publication date.
- Commentaries. These are articles that critically analyze any of the following: (1) policy trends related to educational measurement, (2) relationship between research and evaluation, and (3) connections between research, policy, and practice.
- 7. Critiques of articles. Constructive comments on articles previously published in PJEM are accepted. It is encouraged that these should stimulate discussions and present ideas or alternatives in print. Authors will be invited to respond to the critique made on their article before publication. When possible, the critique and the response will be published at the same time.

Preparation of Manuscripts

Authors are encouraged to prepare manuscripts following the Publication Manual of the American Psychological Association (APA) 5th edition. Manuscripts, including the Abstract and References, should be typed double-spaced on clean short 8th/₂ by 11 inch white bond papers with a margin of one inch on all four sides, using 12-point Times New Roman font and justified to the left margins only. Page numbers should be placed at the center-bottom of the page. Notes, if applicable, are grouped in one section at the end of the article. If the manuscript is based on a thesis or dissertation, a funded research project, or a paper presented at a conference (whether local or international), a footnote on the cover sheet should provide the relevant facts, including the thesis or dissertation adviser or the organization sponsoring the project or conference.

- Author Identification. Since the review process is blind, the first
 page should indicate the title of the article, full name(s) of author(s),
 particulars of present position(s)/institutional affiliation(s), e-mail
 address(es), phone and fax number(s), and any information of
 special relevance. Identity of the author(s) should appear only on
 this page.
- Abstracts and Keywords. All manuscripts must include abstracts of 120 words or less. An abstract is a complete, condensed summary of the article and <u>not</u> a description nor an introduction to the article. In addition, supply 4-5 keywords/phrases that characterize the content of the paper, which can be used for indexing purposes.
- Length of Articles. Manuscripts for theoretical studies, evaluation studies, research studies, and specialized reviews should not exceed 50 pages. Book reviews, commentaries, and critiques of articles should not exceed five pages.
- Language. Manuscripts written in English and Filipino are accepted. However, an English title should be submitted if the manuscript is written in Filipino.
- 5. Tables. Each table should be presented on separate pages and not in the body of the text. It should include a caption and presented in the order in which they appear in the text. As such, they should be given sequential Arabic numbers (i.e., Table 1, etc.), and should be in Microsoft Word (.doc) format. When preparing tables in Microsoft Word, be sure to use the table feature of the program. Furthermore, equations should be generated directly in the text file using the equation feature of Microsoft Word. Importing equations into the text file from a different word processing or graphic applications is discouraged.
- 6. Figures. Illustrations, such as diagrams, drawings, graphs, maps, and photographs, are considered as figures and should be designated as 'Figure 1,' 'Figure 2,' etc., and in sequential Arabic numbers. The text document should not contain the figures but rather are submitted on separate pages at the end of the manuscript and should

be in Tagged Image File Format (TIFF) at a minimum of 600 dpi resolutions. Figure captions should be typed on a separate page, not on the figures, and presented in the order in which they appear in the text.

Since authors know best what they want to show in a figure, they should crop their own figures, leaving only essential materials. If necessary, figures can be rotated 90 degrees and printed sideways. Photocopies of figures are not acceptable.

7. References. References cited in the text document must appear in the reference list at the end of the article. Authors must check that reference details are correct, complete, and in accord with the citations in the text. Authors are responsible for all information in their reference list. The Publication Manual of the American Psychological Association (APA) should be consulted in preparing the reference list. In particular, use of its Fifth edition is highly preferred.

Submission of Manuscripts

Interested contributors must submit the following: (1) one clear printed version of the manuscript, and (2) an electronic copy which may be sent as e-mail attachment to <u>cemresearch@cem-inc.org.ph</u> or on a CD-RW containing the appropriate files.

The electronic copy should follow the style guidelines indicated above (with appropriately placed notes on where to insert the tables and figures in the text). In particular, the following sections of the manuscripts should be submitted as separate files: (1) the title of the article and the abstract, (2) the main text, (3) tables, (4) figures, and (5) references. Appendices are not encouraged but may be allowed if considered necessary to facilitate understanding of the manuscript content. Moreover, authors should submit their figures and tables as camera-ready copies, using a laser-printed computer output, at the maximum dimensions given below.

Paper Orientation	Figure Dimension (in cm)	
	Width	Height
Portrait	6.75	9.0
Landscape	9.0	6.75

The minimum acceptable height for a letter or number in a cameraready figure is about 3 mm or about 9 points. However, it is always better to use larger characters since the Editor may decide to reduce a figure.

Author(s) will be notified in case there is a problem with the electronic files. The printed version of the manuscript, CD-RW copy, and a cover letter should be addressed to:

> The Research Division Center for Educational Measurement, Inc. 24th Floor, Cityland Pasong Tamo Tower 2210 Chino Roces Avenue, Makati City

The Editor will acknowledge receipt of all contributions. The review process is greatly facilitated when manuscripts are submitted in the proper form.

Editorial Procedures

All manuscripts undergo preliminary screening by the Editorial Staff. They determine whether or not to reject the submission outright. If the manuscript fails to meet the Journal's technical and stylistic requirements, it is returned to the author for revision before forwarding to at least two Editorial Consultants for their review. Since the review process is blind, articles sent for review are anonymous. Review is normally completed within three months of the submission, and a comment sheet is provided to facilitate the review. Reviewed manuscripts are generally returned to the authors with specific comments from the Editorial Consultants. Authors may be advised to resubmit their manuscripts to include editorial changes or to submit to an affiliate journal.

The Editor reserves the right to make editorial changes of nonsubstantive nature. Once the final version of the paper has been accepted, authors cannot make further changes to the text. Proofs will be sent to the author(s) if there is sufficient time to do so. These should be corrected and returned to the Editor within seven (7) days to facilitate printing.

Copyright and Permissions

Obtaining necessary permissions for the use of published materials (e.g., figures, tables, etc.) is the responsibility of the authors. These should be properly cited in the list of references. Papers accepted become the copyright of the Journal, unless otherwise specifically agreed, and may not be reproduced without the permission of the Center for Educational Measurement, Inc.

Copies and Honoraria

Authors whose manuscripts are chosen for publication receive a modest honorarium after the article is printed. Additionally, authors receive two complimentary copies of the PJEM issue in which their article appears.

B. The CEM Standard

Description

The CEM Standard is a newsletter that aims to provide a forum for documentation of experiences and best practices at the school and classroom levels, which have direct application for teachers, guidance counselors, and school administrators. The newsletter also addresses issues, questions, and concerns about educational testing and assessment.

Article Content

Articles submitted in The CEM Standard may run from 15 to 50 pages and may fall into the following categories:

- Reflective Documentation. These are narratives or documentation of experiences and/or best practices in the school setting, which reflect the assumptions, importance, and relationship of educational measurement in the teachinglearning process or in school administration.
- Essays. These are short articles that discuss new knowledge or provide insight in the field of educational measurement.

Submission of Manuscripts and Editorial Procedures

The preparation, submission, and editorial procedures followed for processing and reviewing a manuscript submitted for consideration to The CEM Standard is essentially the same as for the PJEM. Thus, the review process may also take up to three (3) months.

Philippine Journal of Educational Measurement

DECEMBER 2008

VOLUME 10 N0. 2

The Philippine Journal of Educational Measurement is published annually by the Center for Educational Measurement, Inc., and contains empirical researches and non-empirical reports such as theoretical notes; research studies; evaluation studies; specialized reviews; essays; reflective inquiry; critical book reviews; commentaries related to educational testing, measurement, assessment, and evaluation; and research on substantive, innovative, and methodological issues.

EDITORIAL STAFF

Maria Cynthia A. Alcantara Editor

> Cecile M. Gatchalian Joy J. Macalintal Editorial Assistants



Center for Educational Measurement, Inc. 24th Floor Cityland Pasong Tamo Tower 2210 Chino Roces Avenue, Makati City Tel. Nos: 813-3691, 813-3694 to 95 Fax Number: 894-1392 www.cem-inc.org.ph